

The influence of initial curvature of the axis upon the member ultimate strength

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SUMMARY

The results of the statistic analysis of load-carrying capacity of struts with relative slenderness values $\bar{\lambda} = 0.7; 1.0; 1.3$ are given in the present paper. The initial curvature of the member axis is modelled by applying the so-called *random fields*. The load-carrying capacity has been determined by geometrically non-linear solution using FEM. The maximum slenderness sensibility to the initial curvature shape modelled by random fields has been analysed. Basic results of experimentally found geometrical and material characteristics of an IPE beam are presented. The random character of input quantities has been taken into consideration in form of histograms. Realizations of random input quantities have been simulated by the numerical simulation method Latine Hypercube Sampling [8].

INTRODUCTION

In general, the axis of a real member is a curve, a totally straight member is practically never concerned. Experimental identification of straight character of a member axis is given, e.g., in [1]. With regard to the fact that the published results of these measurements are not detailed enough and do not reflect the randomness of the phenomenon mentioned to a satisfactory extent, the measurement realizations of the initial curvature of the axis were simulated, in our study, randomly with application of the so-called random fields.

In general, the strut load-carrying capacity is a random quantity. With the aim of approximating the real strut with maximum fidelity, it is necessary to consider the influence of all imperfections separately (yield strength, geometrical imperfections, etc.). The numerical simulation methods of the Monte Carlo type which enable to cover the rich variability of problems can be applied with advantage. In each run, the load-carrying capacity is calculated by non-linear incremental FEM calculation with consideration both of initial curvature of the strut axis, and of the other imperfections. Input random data were obtained [1, 9] or verified experimentally [2, 7].

RANDOM FIELD OF INITIAL CURVATURE OF THE AXIS

Let us imagine the possibility of measuring the initial curvature of the axis of a real beam on pre-selected points, e.g., in the tenths of its length. For these 11 random quantities, an example of the realization of a random field is presented, in magnified scale, in Fig. 1. The form of initial curvature of the member axis is substituted by a cubic spline running from node 1 to node 11. Let us consider the displacement y_i of each i -th node in direction of axis y to be a random quantity with Gaussian distribution, the pre-selected positive correlation being introduced between deviations of neighbouring nodes, so that unreal forms were excluded.

The notion "statistical dependence" can be defined as such a quantitative dependence of one quantity on the other, where also the partial (conditioned) course of corresponding values of the other quantity is determined by the given value of one quantity. The co-variation matrix member $c_{i,j}$ can be determined, e.g., according to the Gaussian auto-correlation function [3, 8]:

$$c_{i,j} = S^2 \cdot e^{-\left(\frac{\xi_{i,j}}{L_{cor}}\right)^2}, \quad (1)$$

where L_{cor} is the so-called correlation length of random field, S is the standard deviation, and $\xi_{i,j} = x_j - x_i$ is the distance between nodes x_i and x_j . The correlation coefficient $\rho_{i,j}$ of the correlation matrix can be determined as:

$$\rho_{i,j} = \frac{c_{i,j}}{\sqrt{c_{i,i} \cdot c_{j,j}}} \quad (2)$$

It will be supposed that with a large number of measurements, the mean value m_{y_i} near to zero would be determined for each random quantity y_i , i.e., it is assumed both positive and negative deviations to occur in the same frequency, and to be, in average, in approximately the same "distance" from the axis of an ideally straight member. For a buckling stressed beam, it will be supposed, further on, that the compression force is located on the straight line passing through the first and the last node of the random field, see Fig. 2. The angle α of rotation of the local co-ordination system with axes x^* , y^* is dependent on the random position of the first and the initial beam node, and can be determined according to the relation:

$$\tan(\alpha) = \frac{\Delta y}{\Delta x} = \frac{y_{11} - y_1}{x_{11} - x_1} \quad (3)$$

The transformation of co-ordinates x , y into the local co-ordination system x^* , y^* can be determined according to the relations:

$$\begin{aligned} y_i^* &= y_i \cos(\alpha) - x_i \sin(\alpha), \\ x_i^* &= y_i \sin(\alpha) + x_i \cos(\alpha). \end{aligned} \quad (4)$$

For $\Delta y \ll \Delta x$ and $y_i \ll 1$ it holds approximately $x_i^* \approx x_i$; this may be a frequent case in practice.

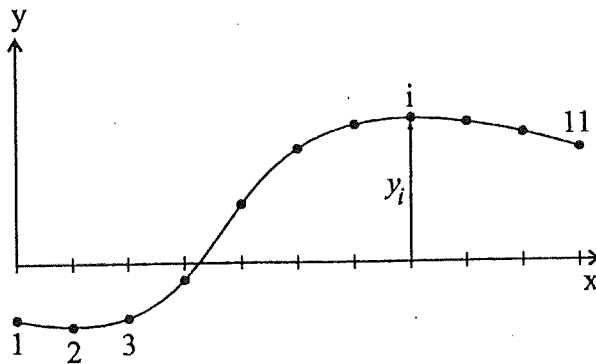


Fig. 1: Random field of beam curvature

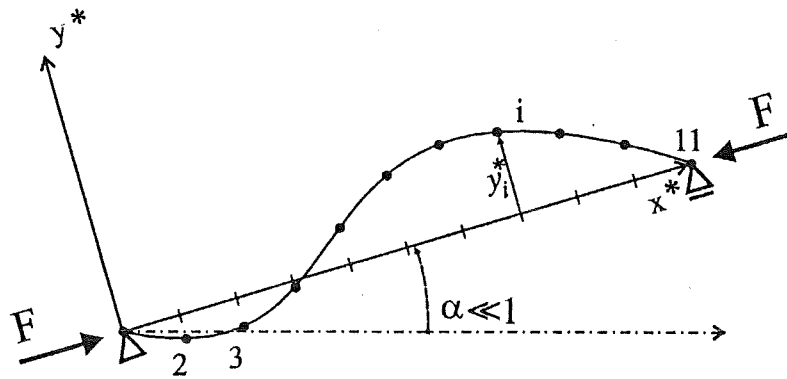


Fig. 2: Local co-ordination system

It can be shown (e.g., by the Monte Carlo method) that the random deviation y_i^* of the i^{th} node in direction of the axis y^* has, after evaluation, the zero mean value $m_{y_i^*} = 0$. The standard deviation value $S_{y_i^*}$ is, due to boundary conditions, zero one in end nodes, its course along the beam length being approximately determined by the sine function the maximum amplitude of which decreases with increasing correlation length. As the angle α is a random quantity as well, is the mutual correlation of quantities y_i^* lower than the correlation of quantities y_i .

STATISTICAL ANALYSIS - STRUT WITH RANDOM CURVATURE

The shape of the beam random curvature is, with regard to the character of beam loading, to be introduced, with advantage, within the local co-ordination system having axes x^*, y^* , see Fig. 2. The random quantities y_i^* will be required to be mutually correlated according to the relation (2). Further on, the sinusoidal course of the standard deviations of quantities y_i^* will be required (5).

The Tolerance Standard [11] presents the maximum allowable deviation of the straightness of I and H bars within 0.1 % to 0.3 % beam length L . In our numerical study, it will be supposed that, when measuring the deviation in the beam half (in the point of expected maximum standard deviation), 95 % of all realizations would be obtained within the interval $\langle -L/1000; L/1000 \rangle$. The maximum deviation is not necessarily to be measured always in the beam half, but it will be of zero value both at its beginning and end. The standard deviation maximum value can be thus derived

according to the rule $2S_x$ by means of value $L/2000$.

$$S_{yi}^* = \frac{L}{2000} \sin\left(\frac{\pi x_i}{L}\right) \quad (5)$$

The beam length L was introduced in a deterministic way, $L = 2\text{m}$. Three variants were solved:

1. Cross-section IPE140; relative slenderness $\bar{\lambda} = 1.3$.
2. Cross-section IPE180; relative slenderness $\bar{\lambda} = 1.0$.
3. Cross-section IPE270; relative slenderness $\bar{\lambda} = 0.7$.

The shape of initial curvature depends on correlation length L_{cor} in the relation (1). Five variants were considered: $L_{\text{cor}}=0.0\text{ m}$; 0.5 m ; 1.0 m ; 2.0 m ; 3.0 m . If the realizations of input quantities are generated by a numerical simulation method of the Monte Carlo type, the value of correlation between individual samples is determined only by order change of samples; it can be found out on behalf of the Spearman rank-order correlation coefficient. Therefore the realization of random field can be generated according to (1) (e.g., with standard deviation $S=1.0$) and subsequently, the standard deviation according to (5) is to be introduced. Realization examples of the beam random curvature are presented in Fig. 3.

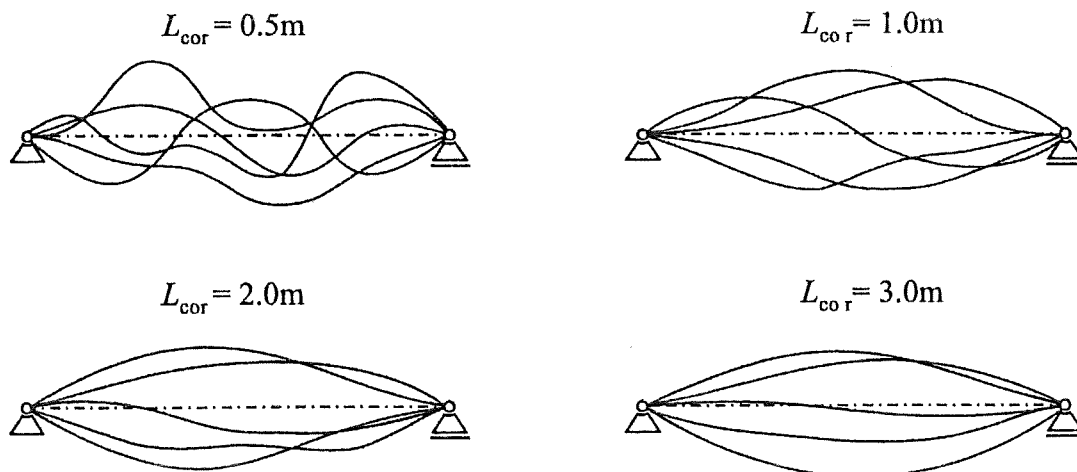


Fig. 3: Example of the beam initial curvature in dependence on correlation length

EXPERIMENTAL RESEARCH – INPUT RANDOM QUANTITIES

Other input random quantities (mechanical and geometrical ones) were considered by means of histograms according to the experimental results of 562 tests of rolled IPE 160 to IPE 220 profiles of steel grades S235, see [2, 7]. The yield strength histogram from samples taken from one third of flange is presented in Fig. 4.

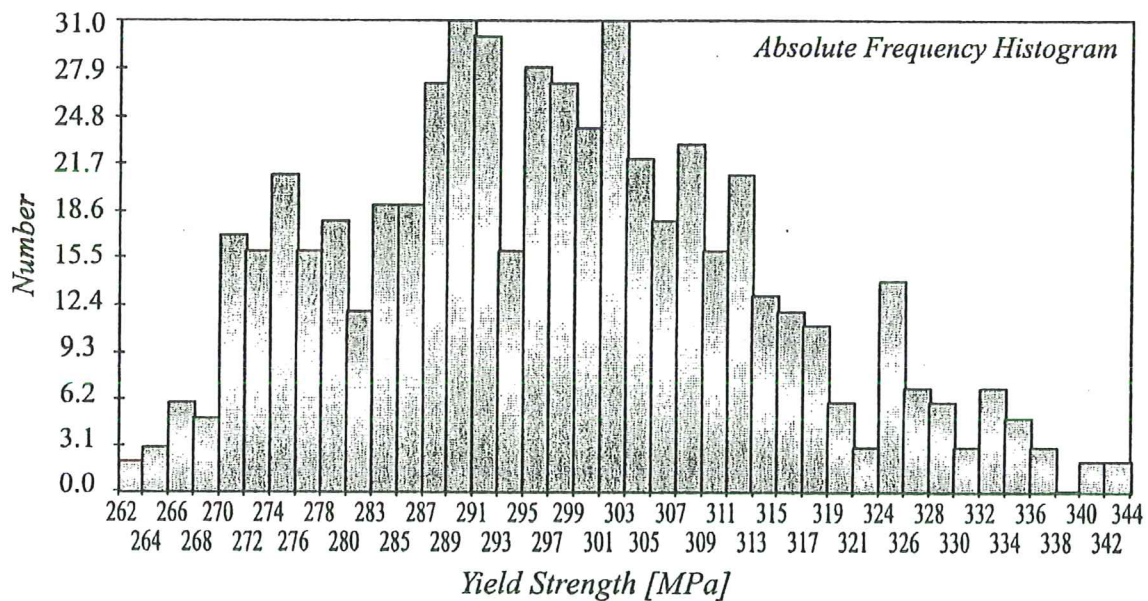


Fig. 4: Yield strength histogram of the steel grade S235

The experimentally found basic statistical characteristics of yield strength, strength and ductility are given in Tab. 1.

Tab. 1: Mechanical characteristics of steel S235 - Valid observations: 562

Quantity	Mean value	Standard deviation	SK Skewness	KU Kurtosis	Minimum value	Maximum value
Yield strength	297.3 MPa	16.8 MPa	0.32462	2.5415	262.0 MPa	344.0 MPa
Tensile strength	403.8 MPa	11.3 MPa	0.32600	2.8447	378.0 MPa	439.0 MPa
Ductility	37.8 %	2.9 %	-0.75266	5.1796	27.1 %	46.2 %

Statistical characteristics of input random quantities the variability of which had to be taken into consideration are presented in Tab. 2. Statistical geometrical characteristics in Tab. 2 are supposed to be relative ones and they have to be multiplied by characteristic values. A biaxially symmetrical cross-section was assumed, i.e., for quantities b , t_2 , histograms obtained by geometry measurements of only one flange were considered. The Young's modulus E was considered, based on two independent experimental results [1, 9] to be a random quantity with mean value $m_E=210$ GPa and with standard deviation $S_E= 12.6$ GPa. An ideally elastic-plastic material was supposed.

Tab. 2: Model of input random quantities

<i>Symbol</i>	<i>Quantity</i>	<i>Type distribution</i>	<i>Mean value</i>	<i>Standard deviation</i>	<i>Skewness</i>	<i>Kurtosis</i>
f_y	<i>Yield strength</i>	<i>Histogram</i>	297.3 MPa	16.8 MPa	0.3246	2.542
h	<i>Cross-sec. height</i>	<i>Histogram</i>	1.001	0.00443	-0.4063	3.015
b	<i>Flange width</i>	<i>Histogram</i>	1.012	0.01026	-0.3939	4.239
t_1	<i>Web thickness</i>	<i>Histogram</i>	1.055	0.04182	1.0545	7.473
t_2	<i>Flange thickness</i>	<i>Histogram</i>	0.988	0.04357	-0.2991	2.663
E	<i>Young's modulus</i>	<i>Gauss</i>	210 GPa	12.6 GPa	0	3

METHODS AND SOFTWARE APPLIED

The load-carrying capacity of an imperfect beam was solved by the finite elements method. A weakly curved beam member having its central line in shape of parabola 3° was applied. The realization of a beam random field is interpolated by cubic spline running from the node 1 up to 11 can be observed in Figs. 1 and 2. Basic functions of beam member are also parabolas of the third degree. The courses of moments are linear, shearing forces show a constant, discontinuous course in nodes, the course of normal force is determined by a parabola 3° .

The load-carrying capacity was calculated by the geometrically nonlinear FEM solution. The beam was divided into 10 beam elements, see Fig. 1. The loading value at the action of which the normal stress in the most stressed beam point will be, with

satisfactory accuracy, equal to yield strength, corresponds to the load-carrying capacity. The Euler - Newton-Raphson method with automatic control of loading step length was applied. The step-by-step halving of loading step is carried out automatically according to the normal stress growth rate in the point of the highest stress value, and according to the determinant value decrease rate. The load-carrying capacity was determined with accuracy of 0.1 %.

When incrementing load, it is necessary to test whether stress has not reached yield strength, namely automatically, member by member, on the whole structure. The determinant value of the tangential stiffness matrix is, in the incrementation method mentioned, always positive. Zero values can be reached, for a slender strut, only in case of an ideally straight beam. However, the probability of perfect beam straightness is, from the point of view of mathematics, of zero value.

The realizations of input random quantities were simulated by the Latin Hypercube Sampling method (LHS) for 200 runs of the method mentioned. The method was described, e.g., in [4, 8]. The method LHS gives more accurate assessments/estimates of mean value and standard deviation for a smaller number of runs, and therefore it is applied, with advantage, in statistical analysis with mathematical FEM models which are of exacting character on computer time [8]. In simulated realizations, the correlations among the quantities y_i^* were determined by order change of samples of random choice on behalf of the Spearman rank-order correlation coefficient.

STATISTICAL ANALYSIS OF LOAD-CARRYING CAPACITY

In Tabs. 3 to 5, there are given the load-carrying capacity statistical characteristics. The statistical evaluation was carried out by means of the programme Statrel 3.1. The design load-carrying capacity was – in compliance with [10] calculated as 0.1 percentile ($\beta_d = 3.8$) both for the Gaussian and lognormal distributions of probabilities. In the column entitled “ ∞ (sin)”, there are given the statistical characteristics of beam load-carrying capacity with primary curvature in shape of sinusoid ($L_{cor} = \infty$) having variable maximum amplitude. Maximum differences among statistical characteristics are presented in the last column of Tabs. 3 to 5. It is to be stressed that, for $L_{cor} = 0.0$ m, the results are influenced by the density of beam division by finite elements, as well.

Tab. 3: Statistical characteristics of load-carrying capacity; $\bar{\lambda} = 0.7$ (IPE270)

	Correlation length L_{cor} [m]						Maximum difference [kN]
	0.0	0.5	1.0	2.0	3.0	∞ (sin)	
m_R [kN]	1192.57	1189.83	1194.35	1202.46	1202.76	1206.56	16.73
S_R [kN]	87.63	95.82	101.51	107.12	105.96	109.15	21.52
0.1 percentile Gauss [kN]	921.79	893.73	880.68	871.43	875.30	869.25	52.54 (6 %)
0.1 percentile lognorm. [kN]	948.07	925.07	915.62	909.98	913.04	909.10	38.97 (4 %)

Tab. 4: Statistical characteristics of load-carrying capacity; $\bar{\lambda} = 1.0$ (IPE180)

	Correlation length L_{cor} [m]						Maximum difference [kN]
	0.0	0.5	1.0	2.0	3.0	∞ (sin)	
m_R [kN]	493.52	472.48	466.60	468.04	466.17	463.69	29.83
S_R [kN]	42.71	49.21	49.49	51.89	50.66	50.66	9.18
0.1 percentile Gauss [kN]	361.53	320.42	313.67	307.67	309.61	307.14	54.39 (16 %)
0.1 percentile lognorm. [kN]	376.49	340.91	334.64	330.58	331.57	329.20	47.29 (13 %)

Tab. 5: Statistical characteristics of load-carrying capacity; $\bar{\lambda} = 1.3$ (IPE140)

	Correlation length L_{cor} [m]						Maximum difference [kN]
	0.0	0.5	1.0	2.0	3.0	∞ (sin)	
m_R [kN]	233.50	225.63	223.39	222.57	222.60	220.81	12.69
S_R [kN]	19.94	20.32	20.72	20.98	20.84	20.63	1.04
0.1 percentile Gauss [kN]	171.91	162.83	159.36	157.75	158.18	157.07	14.84 (9 %)
0.1 percentile lognorm. [kN]	178.79	170.22	167.10	165.71	166.04	164.84	13.95 (8 %)

CONCLUSION

It is evident from the results presented in Tabs. 3 to 5 that, in case of the beam with relative slenderness $\bar{\lambda} = 1.0$, the design load-carrying capacity determined according to [10] as 0.1 percentile is the most sensitive to the central line shape change.

The minimum design load-carrying capacity was determined for beams with primary curvature having the sinusoidal shape ($L_{cor} = \infty$). This fact has proved the more conservative character of stochastic models for which initial curvature is considered in the shape of the sine function. By more accurate modelling of initial central line shape, higher values of design load-carrying capacities can be obtained which will, to the largest extent, manifest themselves in case of beams having relative slenderness approximately $\bar{\lambda} \approx 1.0$.

In beams with relative slenderness $\bar{\lambda} = 0.7$; $\bar{\lambda} = 1.3$, the central line shape variability influenced the load-carrying capacity to a relatively lesser extent. The load-carrying capacity variability of less slender members is the most sensitive to the yield strength f_y and cross-section area A variability [5]. In very slender struts, the load-carrying capacity is, to a limit extent, approximating to the Euler critical force $\pi^2 EI / L^2$, and therefore it is more sensitive to the variability of the moment of inertia I , Young's modulus E and, as the case may be, to beam length L .

A whole series of other stochastic phenomena can be modelled by random fields. In the paper [3], random fields were applied for modelling the yield strength variability along the cross-section. The differences among load-carrying capacities defined according to [10] as 0.1 percentile represented even 10%! In the contribution [6], the influence of material loss by corrosion upon the member ultimate strength was modelled by means of random fields. The influence of the initial curvature shape of the beam central line on fatigue crack initiation and propagation is being studied experimentally.

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