SIMULATION OF RADIOSONDE FLIGHT

Jouni Freund
Mika Reivinen

Rakenteiden Mekaniikka, Vol. 30
Nro 2, 1997, s. 88-100

ABSTRACT

Measurement of the wind velocity and properties of the air by a radiosonde is affected, besides gusts, by the mechanical properties of the system. The dependence of the motion on geometry and other quantities related to the radiosonde can be studied by using a relatively simple model consisting of a ball, a radio and a string between them. The governing equations and the finite element method for the numerical treatment are presented. Also, preliminary numerical results are compared with experimental ones.

INTRODUCTION

Figure 1 shows schematically the components of a radiosonde. The system, consisting of a radio, a ball and a string, is released at the ground level. During the take-off phase the string is unrolled from the coil attached to the ball to its final length. After that the system continues to rise until the ball explodes at the height of about $30\ \mathrm{km}$. Experimental results show that the terminal speed $(5...6)\ \mathrm{m/s}$ in the vertical direction is almost independent of the height of the ball meaning that the time to reach the final height is about 1.5h. During the flight the radio part transmits continuously information about the atmospheric properties and the air velocity assuming that the acceleration of the radio does not exceed a certain limit.

The external forces acting on the radiosonde are gravity, buoyancy and aerodynamic force. The buoyancy force is present also when the sonde is at rest relative to the air. Then the aerodynamic part is, however, zero. To simplify the setting, we assume that the wind flow field is given and that the ball can be taken as a rigid sphere when considering the fluid flow around it. Actually the drag by the relative motion of the ball with respect to the air produces flattening and the true shape of the ball is something like that of an egg whose sharper end is attached to the string. Also the string force may distort the shape.

According to Sakamoto & Haniu (1990), one may observe various flow patterns around a spherical object depending on the Reynolds number. In the interesting range of Reynolds number ($Re \approx 10^6$) the flow behind the sphere is more or less irregular which means that the aerodynamic force acting on the ball should also be irregular without noticeable periodic components. This effect is taken into account by introducing an irregular component, representing also gusts, into the flow field.

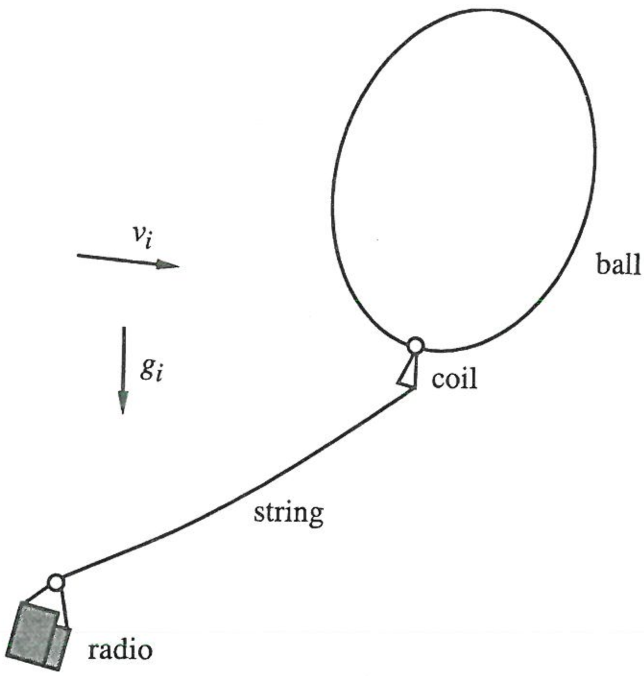

Figure 1. The main components of a radiosonde.

GOVERNING EQUATIONS

In the model the coil is accounted for as an interface condition between the ball and the string. The ball and the radio are treated as rigid bodies. The string is taken to be flexible and almost inextensible. Furthermore, the equations of the motion of the ball, the radio and the string are written directly in the inertial coordinate system whose origin is placed on the earth surface in such a way that the x_1- and x_2-axes are tangents to the surface and the x_3-axis points upwards.

Let us recall some of the kinematic formulas needed in the treatment of rigid bodies. In what follows similar notations are applied for both the vector components and the coordinates of a point. The well known transformation formulas relating the quantities of the body fixed and inertial coordinate systems are

$$
\begin{aligned}
x_i &= x_i^0 + y_i \ , \\
\dot{x}_i &= \dot{x}_i^0 + e_{ijk}\omega_j \, y_k \ , \\
\ddot{x}_i &= \ddot{x}_i^0 + (e_{ijk}\dot{\omega}_j + e_{ijm}e_{mlk}\omega_j\omega_l) y_k \ ,
\end{aligned}
\tag{1}
$$

where $y_i = T_{ij}\,\xi_j$, $e_{ijk} = (j-i)(k-j)(i-k)/2 \quad i,j,k \in \{1\ldots3\}$ is the permutation symbol, the superimposed dot means derivative with respect to time, x_i^0 is the position vector of the origin of the body fixed coordinate system, ξ_i denotes the position vector of a generic material point in the body fixed coordinate system and y_i the same vector in the inertial coordinate system, x_i denotes the position vector of a generic material point in the inertial coordinate system, ω_i is the angular velocity of the body represented in the base of the inertial coordinate system and T_{ij} is the transformation matrix.

The transformation matrix defining the angular position of a rigid body can be written in terms of the Euler parameters $\varepsilon_i \ \ i \in \{1\ldots4\}$. The parameters are not independent as the sum of their squares is unity. Expression

$$
[T_{ij}] = \begin{bmatrix}
\varepsilon_1^2 - \varepsilon_2^2 - \varepsilon_3^2 + \varepsilon_4^2 & 2(\varepsilon_1\varepsilon_2 - \varepsilon_3\varepsilon_4) & 2(\varepsilon_1\varepsilon_3 + \varepsilon_2\varepsilon_4) \\
2(\varepsilon_1\varepsilon_2 + \varepsilon_3\varepsilon_4) & -\varepsilon_1^2 + \varepsilon_2^2 - \varepsilon_3^2 + \varepsilon_4^2 & 2(\varepsilon_2\varepsilon_3 - \varepsilon_1\varepsilon_4) \\
2(\varepsilon_1\varepsilon_3 - \varepsilon_2\varepsilon_4) & 2(\varepsilon_2\varepsilon_3 + \varepsilon_1\varepsilon_4) & -\varepsilon_1^2 - \varepsilon_2^2 + \varepsilon_3^2 + \varepsilon_4^2
\end{bmatrix}
\tag{2}
$$

is given for example in Huston (1990). The differential equations relating the angular velocities and the Euler parameters are

$$
\begin{bmatrix} \omega_1 \\ \omega_2 \\ \omega_3 \\ 0 \end{bmatrix}
= 2 \begin{bmatrix}
\varepsilon_4 & -\varepsilon_3 & \varepsilon_2 & -\varepsilon_1 \\
\varepsilon_3 & \varepsilon_4 & -\varepsilon_1 & -\varepsilon_2 \\
-\varepsilon_2 & \varepsilon_1 & \varepsilon_4 & -\varepsilon_3 \\
\varepsilon_1 & \varepsilon_2 & \varepsilon_3 & \varepsilon_4
\end{bmatrix}
\begin{bmatrix} \dot{\varepsilon}_1 \\ \dot{\varepsilon}_2 \\ \dot{\varepsilon}_3 \\ \dot{\varepsilon}_4 \end{bmatrix}
\equiv 2[E_{ij}]\{\dot{\varepsilon}_j\} \ .
\tag{3}
$$

The first three equations can be used to eliminate the angular velocities from the equations of motion to be presented shortly. Let us note that the kinematical matrix E_{ij} is orthogonal which means that its inverse coincides with its transpose.

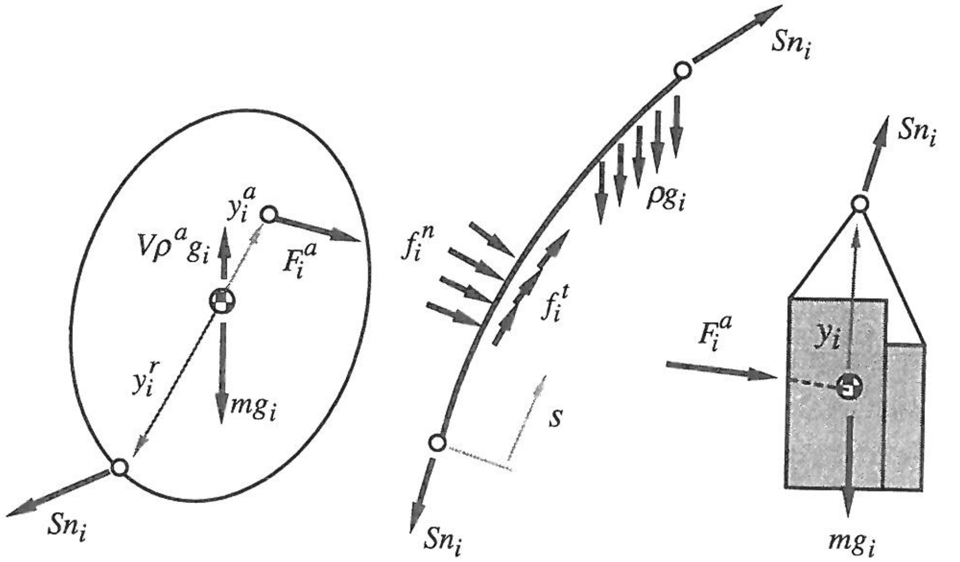

Figure 2. The main components of a radiosonde and the external and internal forces. The mass center of the radio is assumed to be located on the line of action of the aerodynamic force.

Ball and radio

The governing equations for the ball and the radio differ only with respect to the expression for the aerodynamic force. So the following discussion is intended for both parts. In the inertial coordinate system the equations of motion are

$$
\begin{aligned}
& m\ddot{x}_i = F_i , \\
& (I_{ij}\omega_j)^{\cdot} = M_i ,
\end{aligned}
\tag{4}
$$

where m is the mass, F_i is the resultant of the external forces, I_{ij} is the inertia tensor with respect to the mass center and M_i is the resultant moment vector of the external forces with respect to the mass center. For simplicity we assume that $I_{ij} = I\delta_{ij}$, where δ_{ij} is the Kronecker delta, although the inertia tensors are actually not isotropic. Also, when considering the rotation of the ball, the mass of the contents (hydrogen) is omitted since it affects the motion mainly by adding a damping component due to the viscosity.

The external forces are due to gravity, pressure differences and viscosity of air. The mass center of the ball, including the coil, and the point of action of the resultant of the aerodynamic force, called the aerodynamic center, do not necessarily coincide. Assuming that the aerodynamic center follows the ball motion, the relative position vector of the aerodynamic center is given by $y_i^a = T_i d$, where d is the signed distance from the mass center and $T_i \equiv T_{i3}$. Similarly one may write $y_i^r = -T_i r$ (Figure 2). We further assume that the point of action of the buoyancy force coincides with the mass center. Then, the external force system reduced with respect to the mass center is

$$F_i = (m - \rho^a V) g_i - S n_i + F_i^a - \Delta m \dot{u}_i \ ,$$
$$M_i = e_{ijk} T_j (d F_k^a - r S n_k) \ , \qquad (5)$$

where $u_i = \dot{x}_i - v_i$ is the relative velocity, g_i is the acceleration by gravity, v_i is the velocity of the air, S is the string force magnitude, n_i is the unit tangential vector to the string at the point of contact, ρ^a is the density of air, V is the volume of the ball and Δm is the added mass taking into account the inertia of the air.

Often the aerodynamic force is taken to consist of the drag component aligned with the relative flow direction and the lift component located in the normal plane to the relative flow direction. As the lift coefficient is somewhat problematic in this particular case, we apply the simple expression

$$F_i^a = -\frac{1}{2} \rho^a A u C_D [u_i \kappa + (1 - \kappa) u_j T_j T_i] \qquad (6)$$

instead. In (6) we have divided the relative flow vector into two components of which one is aligned with T_i (the symmetry axis of the ball) and the other is located in the normal plane to T_i. After that the drag force components in these directions are calculated from a simple formula for a sphere (obtained by the selection $\kappa = 1$). The drag coefficient is given in Schlichting (1974) as function of the Reynolds number and the adjustable parameter κ denotes the relative value of AC_D in the two directions explained above. In the writers opinion, strive for a more detailed formula is not reasonable, because the actual shape of the ball can not be predicted in any simple way.

String

The equation of motion of the string written in the inertial coordinate system is

$$\rho \ddot{x}_i = (S n_i)_{,s} + f_i \ , \tag{7}$$

where $n_i = x_{i,s} / \sqrt{x_{j,s} x_{j,s}}$ is the unit tangential vector to the string, S is the string tension, f_i is the external force vector per unit length, ρ is the density per unit length and s denotes the coordinate having the physical meaning of the string arc length. We assume constitutive relation

$$S = E \cdot (\sqrt{x_{i,s} x_{i,s}} - 1) \tag{8}$$

between the string force magnitude and the elongation of the string. Equation states that the string force is proportional with factor E to the relative elongation.

The buoyancy force on the string will be omitted as negligible. The external force vector per unit length comes from gravity, from pressure differences, viscosity of air and from the added mass term. Since the term due to viscosity is somewhat different in the tangential and normal directions to the string, the following expression is applied:

$$f_i = \rho g_i - k\pi D \mu u_i^t - C_D \frac{1}{2} D \rho^a u_i^n \| u_j^n \| - \Delta\rho \dot{u}_i^n \ , \tag{9}$$

where $u_i^t = n_i n_j u_j$ and $u_i^n = u_i - u_i^t$ ($u_i = \dot{x}_i - v_i$), ρ is the mass of the string per unit length, μ is the viscosity of air, D is the diameter of the string and k is the boundary layer parameter. The drag coefficient C_D and the added mass $\Delta\rho$ are discussed for example in Blevins (1990).

INTERFACE CONDITIONS

As the ball, string and the radio are not allowed to move totally free with respect to each other, some kinematical conditions at the contact points are needed. The radio-string interface condition states that the relative displacement is zero. The condition restricting the relative motion of the ball and the string is more complicated since the string is unrolled to its final length during the first minute of the flight. Anyway, as we assume that the string is unrolled with a constant speed, both interface conditions can be written as

$$[\![\dot{x}_i n_i]\!] = \dot{a} \;,\;\; [\![(\delta_{ij} - n_i n_j)\dot{x}_i]\!] = 0 \,, \tag{10}$$

where $\dot{a}$ is the given unrolling speed and the jump bracket is defined by $[\![x_i]\!] = x_i - \bar{x}_i$, where the overbar means a quantity known from initial or boundary conditions. The equations state that the speed of the string measured from the ball (the coil is attached to the ball) is $\dot{a}$ in the direction of the string and the relative velocity is zero in the normal plane to that direction. At $s = 0$ the two first conditions can be replaced by $[\![x_i]\!] = 0$.

NUMERICAL METHOD

The time-discontinuous Galerkin method, where the initial and interface conditions are forced in the weak sense, is applied to solve the system of equations numerically. This introduces some complications in the variational problem, but simplifies the overall solution scheme considerably. For detailed explanations of how to proceed in practice we refer to Freund (1996).

One may think that in the numerical method the radio-string-ball system is treated as a discrete system shown schematically in Figure 3. In the figure the nodes (small circles numbered from 1 to 6) represent the points where the discrete values of the unknown functions are sought. Since the string forms a continuum, the unknowns are interpolated between the nodes. In conventional terms of the finite element method the string is divided into linear two-noded elements (in the spatial dimension). Also, the ball and the radio are considered as two-noded elements. The way to end up with the discrete system is discussed next.

Finite element spaces

The finite element space for the rigid bodies consists of piecewise continuous polynomials of degree p. In a typical time interval $t \in \{\bar{t}, \bar{t} + \Delta t\}$, where Δt is the step size, the definition reads

$$U = \{u : u = \textstyle\sum_{i \in \{0 \ldots p\}} u_i (t - \bar{t})^i / i! \text{ and } u_i \in \mathrm{R}\} \,, \tag{11}$$

where the constants u_i denote derivatives of order i with respect to time at $t = \bar{t}$. The other space needed for the treatment of the string part of the system is

$$V = \{u : u = \sum_{i \in \{0 \ldots p\}} u_i (t - \bar{t})^i / i! \text{ and } u_i \in H^1(\Gamma_-)\}, \tag{12}$$

where the coefficients are now functions not constants (actually piecewise linear in s) and Γ_- denotes the part of the boundary of the space-time domain where the initial conditions are given or where the solution is known from the previous interval. Let us note that the set applied in connection with the rigid bodies is actually also (12) where we, however, force the nodal parameters to take the same values at each node. Then, what is left is (11).

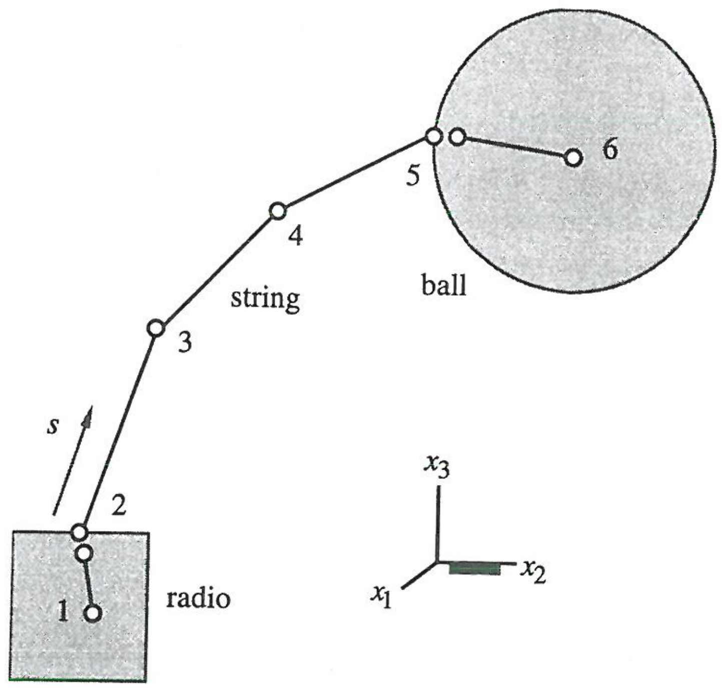

Figure 3. Discretization of the radio-string-ball system. The actual number of the nodes used in the calculations is larger than the one given in the figure.

Ball and radio

The unknowns of the ball problem are the position of the mass center and the Euler parameters. The numerical problem reads: find $x_i \in U \;\; i \in \{1 \ldots 3\}$ and $\varepsilon_i \in U \;\; i \in \{1 \ldots 4\}$ such that

$$(w_i, m\ddot{x}_i - F_i) - \langle w_i m, [\![\dot{x}_i]\!] \rangle + \langle \dot{w}_i m, [\![x_i]\!] \rangle +$$
$$+(z_i, I E_{ij}\ddot{\varepsilon}_{ij} - M_i) - \langle z_i I E_{ij}, [\![\dot{\varepsilon}_j]\!] \rangle + \langle \dot{z}_i I E_{ij}, [\![\varepsilon_j]\!] \rangle = 0 \tag{13}$$

for all the test functions $w_i \in U \;\; i \in \{1\ldots3\}$ and $z_i \in U \;\; i \in \{1\ldots4\}$. The notations $(\cdot,\cdot)$, $\langle\cdot,\cdot\rangle$ denote the $L_2(\mathrm{T})$ and $l_2(\partial\mathrm{T})$ inner products ($\partial\mathrm{T}$ consists of the initial time and the interfaces of the time intervals).

The fourth moment equation associated with ε_4 is actually a constraint by which the sum of the squares of the Euler parameters is forced to unity. As the differentiated form may lead to error accumulation and, finally, to a value other than one, the constraint is added to the fourth equation and the differentiated form of it is multiplied by $\tau^2 I$ with $\tau = \alpha\Delta t$, where α is chosen small enough to prevent oscillations in the numerical solution.

String

For practical reasons the originally non-rectangular space-time solution domain Ω is mapped onto rectangle $\Omega' = [0,1]\times\mathrm{T}$ by using the transformation $s = s'a(t)$, where $a(t)$ is the true length of the string and $s' \in [0,1]$. The derivatives of function x_i in the two systems are related by $x_{i,s} = x_{i,s'}a^{-1}$ and $x_i' = \dot{x}_i - x_{i,s'}\dot{a}s'a^{-1}$.

The numerical problem corresponding to the time-discontinuous Galerkin method and weakly enforced boundary conditions reads: find $x_i \in V \;\; i \in \{1\ldots3\}$ such that

$$(w_i, \rho x_i'' - f_i) - \langle w_i \rho n_t, [\![x_i']\!] \rangle + \langle w_i' \rho n_t, [\![x_i]\!] \rangle + \Delta t^{-1} \langle w_i \rho n_t n_t, [\![x_i]\!] \rangle +$$
$$+ (w_{i,s}, \overline{S} x_{i,s}) - \langle w_i, \overline{S} x_{i,s} n_s \rangle + \langle w_{i,s} \overline{S} n_s, [\![x_i]\!] \rangle + h^{-1} \langle [\![w_i]\!] \overline{S} n_s n_s, [\![x_i]\!] \rangle = 0 \tag{14}$$

for all the test functions $w_i \in V \;\; i \in \{1\ldots3\}$. The notations $(\cdot,\cdot)$ and $\langle\cdot,\cdot\rangle$ mean the $L_2(\Omega)$ and $L_2(\partial\Omega)$ inner products and $\overline{S} = S/\sqrt{x_{i,s}x_{i,s}}$. The quantities n_i and n_s, n_t (n_t is the absolute value of the component) are not related despite similar notation. The former denote the tangent of the string in the physical space and the latter are the components of the unit outward normal vector to the solution domain.

Remark. The reason for using weakly enforced interface conditions is the form of the ball-string interface condition, which causes difficulties if one tries to enforce it strongly, whereas the selection where the rigid body motion is affected by the string force and the string motion by the position of the contact point finds its explanation in the physics of the problem.

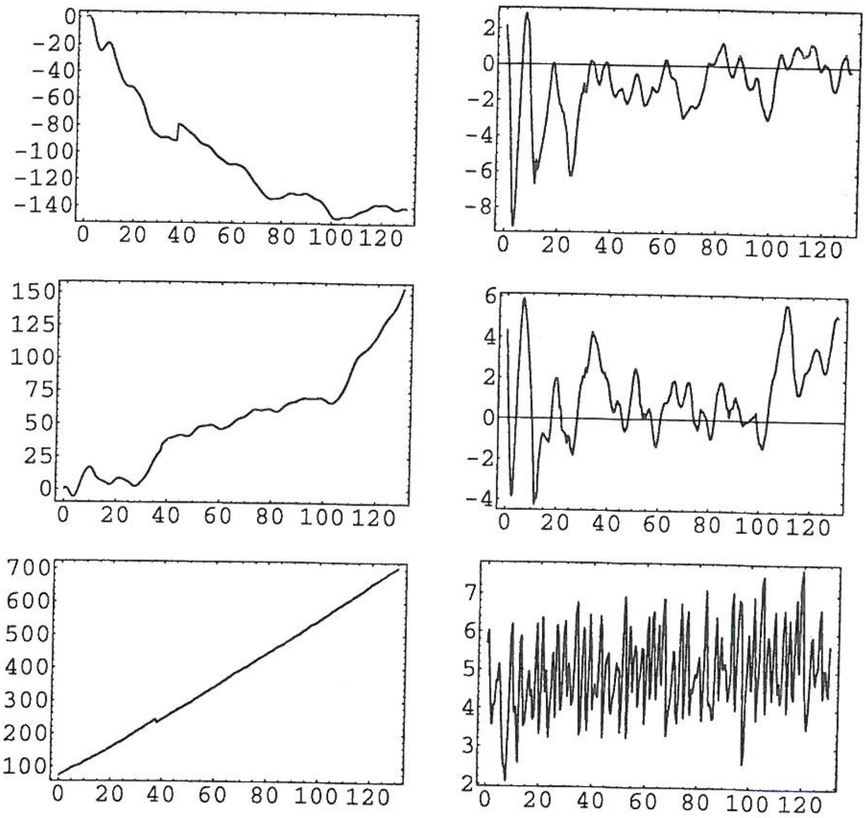

Figure 4. Experimental results for the radio. In the first, second and third row $x_1, \dot{x}_1$, $x_2, \dot{x}_2$ and $x_3, \dot{x}_3$, respectively.

NUMERICAL RESULTS

A FORTRAN90 program (called FEAP) developed in the laboratory of Computational Dynamics by the first writer of this article, was used in the numerical calculations. The preliminary version is explained in Freund & Lempinen (1994) and a more detailed discussion of the underlying theory is given in Freund (1996). The program is designed to handle problems expressible in the space-time variational form without particular restrictions on the number of unknowns or problem dimension. The main assumption is that the variational form works with similar approximations for all the unknowns. The present problem was added to the problem set of FEAP with the name *radiosonde* having a subset containing problems *string*, *ball* and *radio*. The objects *string*, *ball* and *radio* can be considered as physical parts that can be joined in any reasonable way (not necessarily in the simple way shown in Figure 3) to produce something called *radiosonde*.

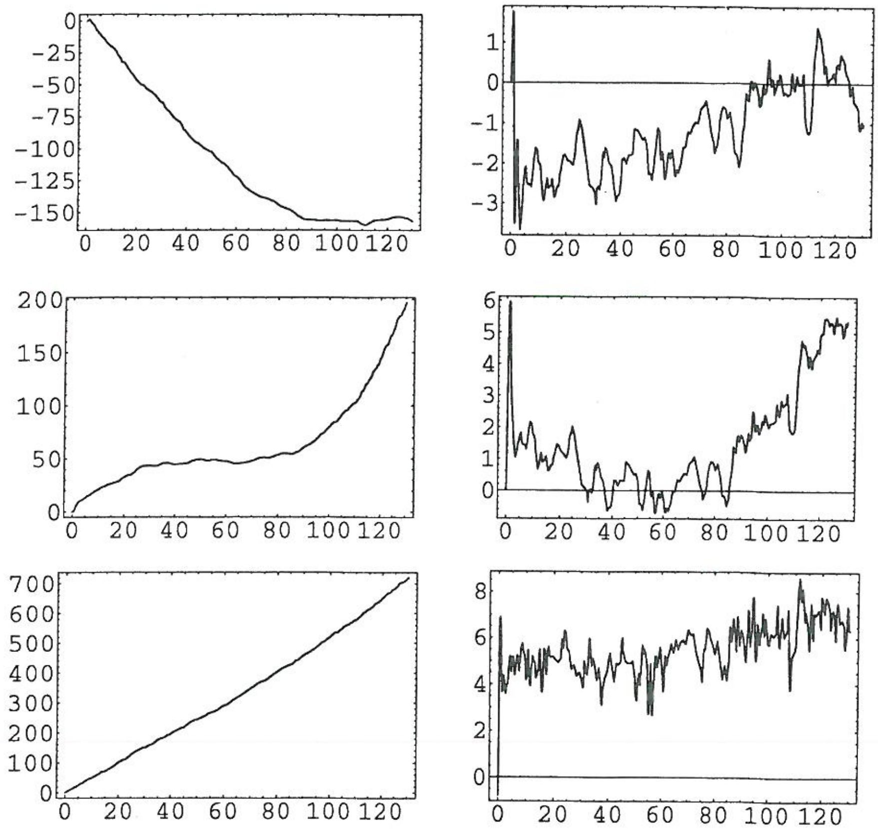

Figure 5. Numerical results for the radio. In the first, second and third row $x_1, \dot{x}_1$, $x_2, \dot{x}_2$ and $x_3, \dot{x}_3$, respectively.

Figures 4 and 5 show the experimental and the numerical results for the radio part during the first two minutes after the take-off. Polynomial approximation of the mean horizontal air velocity extracted from Figure 4 is given by

$$v_1 = \left[-2.8 + 1.0 \cdot z' + 22.2 \cdot (z')^2 - 26.6 \cdot (z')^3\right] \mathrm{m/s},$$
$$v_2 = \left[1.4 - 8.1 \cdot z' + 16.6 \cdot (z')^2 + 1.4 \cdot (z')^3\right] \mathrm{m/s}, \quad (15)$$

where $z' = z / z_1$ and $z_1 = 1000\text{m}$. Although (15) is a reasonable approximation for the mean air velocity, one can not expect more than similarity between the experimental and numerical results since the initial conditions are known only roughly, the vertical speeds do not coincide and the gustiness can be modelled only in the average sense. In fact the present gust model makes use of random sampling of frequencies from a certain range with the consequence that details of the solution vary from simulation to simulation. More detailed discussion is given in Freund & Reivinen (1997).

CONCLUDING REMARKS

It seems that acceleration experienced by the radio part depends highly on the wind model used. Consequently a reasonable model for gusts is needed to make the numerical results similar to the experimental ones. Another point that needs attention is the aerodynamic force to the ball. In our opinion the question why the vertical speed of the ball remains almost constant despite changes of almost all the quantities affecting the motion can not be answered without more sophisticated models for gustiness and the aerodynamic force. The present model serves well, however, for studies on how the problem parameters affect the radiosonde motion.

ACKNOWLEDGMENTS

This work has been supported by Vaisala OY. The writers are grateful to associate professor Eero-Matti Salonen for his valuable comments during the writing process. Also, Timo Saarnimo from Vaisala OY deserve many thanks for useful discussions, information concerning the structure of a radiosonde and experimental results.

REFERENCES

Blevins D.B., Flow-Induced Vibration, Van Nostrand Reinhold, 1990.

Freund, J., Lempinen, A. A general purpose finite element solver, Report 40, Helsinki University of Technology, Faculty of Information Technology, Laboratory of Computational Dynamics, 1994.

Freund, J., Space-Time Finite Element Methods for Second Order Problems: an Algorithmic approach, Acta Polytechnica Scandinavica, Ma 79, 1996.

Freund, J., Reivinen, M., Simulation of radiosonde flight, Report, Helsinki University of Technology, Department of Mathematics and Systems Analysis, Laboratory of Computational Dynamics, 1997. In preparation.

Huston, R.L., Multibody Dynamics, Reed Publishing inc., 1990.

Sakamoto, H., Haniu, H., A Study on Vortex Shedding From Spheres in a Uniform Flow, ASME Journal of Fluids Engineering, Vol.112, December 1990.

Schlichting, H., Boundary Layer Theory, McGraw-Hill, 1974.

Jouni Freund
Mika Reivinen

Helsinki University of Technology
Department of Engineering Physics and Mathematics
Laboratory of Computational Dynamics