WATER COLUMN MOTION IN A MOONPOOL OF A SHIP

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Abstract

A method of evaluating a motion of a water column in a moonpool of a ship is presented. This motion is caused by irregular waves that are encountered by a ship. Two cases are dealt with. The first one is an open moonpool with atmospheric pressure at the free water surface. The second case is a moonpool covered by a hermetic cover. The results of numerical simulations are compared with the results of model scale experiments. An application example of the method is presented.

Introduction

In drilling and in some research ships a vertical well is often used. Well extends from the ship bottom to the weather deck. This well is commonly called moonpool. It is used for lowering the research or drilling equipment from a ship to the sea. Vertical motion of a water column in a moonpool of a ship caused by waves may be a cause of ship's operability limitations. In an extreme case this motion leads to water overflow that may cause stability risk of a ship or damage to the deck equipment. In order to prevent overtopping the moonpool can be covered by a hatch. However, this is associated with significant air-pressure loads on the hermetic cover.

Aalbers (1984) presented a theoretical model of the water motion in an open moonpool of a floating vessel. He derived the governing equations applying the integral form of the momentum conservation principle. A numerical method for evaluating vertical motion of a water column in the moonpool of a research vessel was also presented by Matusiak (1986). The non-linear ordinary differential equation for the water column motion in a stationary ship was derived using the second order Lagrange equation. Simulated motion compared fairly well with the model tests' results. The method was also used to evaluate water column motion in a hermetically covered moonpool. Air pressure loads acting on a cover were also evaluated numerically.

In the following a theoretical model for evaluating vertical motion of a water column in a moonpool is presented. Location of a moonpool in a floating ship is shown in figure 1.

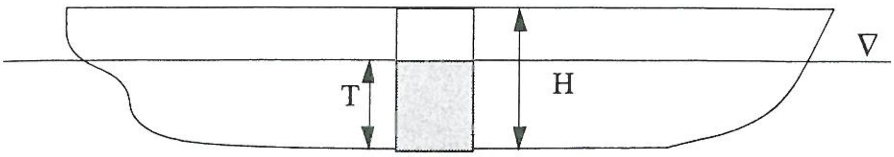

Fig. 1 Moonpool in a ship.

The method is based on the integral form of the momentum conservation principle as in the method of Aalbers (1984). However, the resulting equation of motion is different. The main reason for this difference is a different formulation of the reaction forces acting on the water column at the ship bottom. Moreover, the effect of a hermetic covering of the moonpool is dealt with.

General assumptions

The flow of water in the moonpool is assumed to be inviscid. Only vertical motion of the water column is considered. That is, transverse motion of the water and sloshing in the moonpool are disregarded.

Wave-induced pressure acting on the water column is assumed to be sufficiently well represented by the Froude-Krylov pressure. In other words diffraction and radiation pressures caused by a ship are disregarded. Waves of a length considerably longer than the ship breadth are considered only. The effect of ship motions on the moonpool behaviour is taken into account.

Theoretical model of the water column motion in a moonpool of a floating ship

The moonpool having vertical smooth walls and a definition of the variables used in the analysis are presented in figure 2.

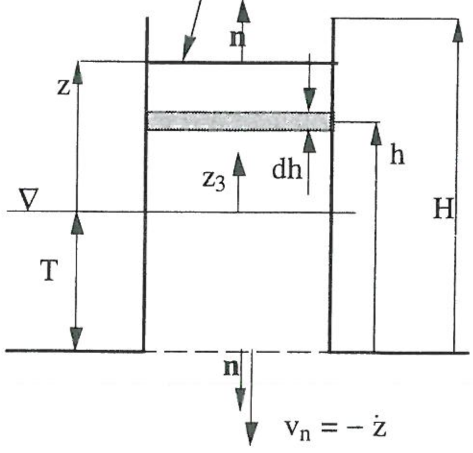

Fig. 2 Moonpool with vertical smooth walls.

In the following we derive equation of motion for the water column motion using the integral form of the conservation of momentum. Two coordinate systems are used. Ship motion that is relevant for the moonpool dynamics is described by the heaving motion z_3 of the ship at the moonpool location. If moonpool is located at the symmetry plane of a ship then this motion is a result of ship heave and pitch motions. Water column elevation is denoted by z and it is measured in the coordinate system fixed to the ship.

Mass conservation

We consider as a control volume the volume V of the water column in the moonpool

$$V(t) = A\,[T + z(t)], \tag{1}$$

where A is the area of the horizontal cross-section of moonpool and T is ship draft.

Mass conservation law can be expressed in the following form (Pnueli, 1992)

$$\frac{D}{Dt}\int_{V(t)} \rho \, dV = \frac{\partial}{\partial t}\int_{V(t)} \rho \, dV + \int_{S_B} \rho \, v_n \, dS = 0, \tag{2}$$

where t is time, ρ water density, V the fluid volume and S_B is the moonpool surface at the ship bottom. v_n is normal to the surface S_B component of the flow velocity. This normal component of the velocity is positive when pointing outwards from the control volume V. Equation (1) means that the rate of change of mass in the control volume V equals the flux of mass into the control volume. For the moonpool having vertical walls normal flow velocity at the bottom is

$$v_n = -\dot{z} = -\dot{z}_B, \tag{3}$$

where $\dot{z}$ is water column velocity and $\dot{z}_B$ is flow velocity at the bottom opening S_B. Applying relations (2) and (3) to the mass conservation law (1) yields

$$\rho\, A \frac{\partial}{\partial t}[T + z(t)] = \rho\, S_B\, \dot{z}. \tag{4}$$

Thus the mass conservation law is fulfilled because $A = S_B$.

Momentum conservation

Momentum conservation law can be expressed as follows (Pnueli, 1992)

$$\frac{D}{Dt}\int_V \rho\, \dot{z}\, dV = \int_S \mathbf{f}\; dS + \int_V \rho\, \mathbf{g}\, dV, \tag{5}$$

where $\mathbf{f}$ is the surface force at the control volume boundaries and $\rho\, \mathbf{g}$ is the body force per unit volume. The left hand side of equation (5), that is total momentum change in the control volume V, can be expressed as follows

$$\begin{aligned} \frac{D}{Dt}\int_{V(t)} \rho\, \dot{z}\, dV &= A \frac{\partial}{\partial t}\int_0^{T+z(t)} \rho\, \dot{z}\, dh - \int_{S_B} \rho\, \dot{z}_B |\dot{z}_B|\, dS \\ &= \rho\, A \frac{\partial}{\partial t}\{[T + z(t)]\,\dot{z}\} - \rho\, S_B\, \dot{z}_B |\dot{z}_B| = \rho\, A\, \ddot{z}[T + z(t)]. \end{aligned} \tag{6}$$

Expression (6) comprises the instantaneous change of momentum and addition of momentum due to free surface motion z(t). This additional term is cancelled by the flux of momentum through the bottom opening S_B.

Forces at the bottom and free water surface of the moonpool

Because of the symmetry condition, the surface forces acting on the control volume surfaces being in contact with the moonpool walls cancel themselves. Thus the surface forces acting at the water column free surface and at the bottom have to be considered only. The forces acting on the water column at the bottom of the moonpool comprise the reaction force F_R due to water flux, force F_D due to dynamic pressure, hydrostatic force F_H, radiation force F_A and Froude-Krylov wave force F_{FK}, i.e.

$$F_{S_B} = \int_{S_B} \mathbf{f}\ dS = F_R + F_D + F_H + F_A + F_{FK}$$

$$= \rho\ S_B \left[-\dot{z}\,|\dot{z}| - \frac{1}{2}(\dot{z})^2 + g\left(T - z_3\right)\right] - A_z\left(\ddot{z} + \ddot{z}_3\right) + S_B\left[p_a + p_{FK}(z,t)\right], \quad (7)$$

where A_z is the added mass of the moonpool cross-section at the ship bottom. It is customary to express added mass in a non-dimensional form that is called *'added mass coefficient'*. We apply a definition of added mass coefficient of the following form

$$C_A = \frac{A_z}{\frac{2}{3}\rho\ \pi\ R^3}, \quad (8)$$

where R is the radius of the circular cross-section or $R = \sqrt{b\ l\ /\ \pi}$ for rectangular cross-section.

For the open moonpool the only surface force acting on the water free surface is the force due to atmospheric pressure p_a, i.e.

$$F_{S_{FS}} = \int_{S_{FS}} \mathbf{f}\ dS = -\ S_{FS}\ p_a\ . \quad (9)$$

If a moonpool with vertical walls is covered hermetically by a cover then air compression due to water column motion has to be taken into account. Assuming air compression to be an adiabatic process the pressure acting on the free surface can be expressed as follows

$$p_{FS} = p_a \left[\frac{H - T}{H - T - z(t)}\right]^{\kappa}, \quad (10)$$

where H is the height of the moonpool and for air adiabatic constant $\kappa = 1.4$. Thus for the hermetically covered moonpool the surface force acting on the free surface is

$$F_{S_{FS}} = \int_{S_{FS}} \mathbf{f}\ dS = -\ S_{FS}\ p_a \left[\frac{H - T}{H - T - z(t)}\right]^{\kappa}. \quad (11)$$

Body forces of the water column

The only body forces acting on a control volume are the gravitational force and the inertia force due to ship heave motion z_3. Thus the body force is

$$\int_V \rho\, g\, dV = -\rho\, A\left(g + \ddot{z}_3\right)\int_0^{T+z} dh = -\rho\, A\left(g + \ddot{z}_3\right)(T + z). \qquad (12)$$

Equation of water column motion

Collecting all terms of the momentum conservation equation (5) the equation governing the water column motion in the uncovered moonpool having vertical smooth walls and cross-section of an arbitrary form is obtained

$$\left(\ddot{z} + \ddot{z}_3\right)\left[T + z + \frac{2}{3} R\, C_A\right] + \dot{z}\left|\dot{z}\right| + \frac{1}{2}\left(\dot{z}\right)^2 + g\left(z + z_3\right) = \frac{1}{\rho} p_{FK}(z,t). \qquad (13)$$

For the moonpool covered hermetically, the equation of water column motion is

$$\left(\ddot{z} + \ddot{z}_3\right)\left[T + z + \frac{2}{3} R\, C_A\right] + \dot{z}\left|\dot{z}\right| + \frac{1}{2}\left(\dot{z}\right)^2 + g\left(z + z_3\right)$$
$$+ \frac{1}{\rho} p_a \left\{\left[\frac{H - T}{H - T - z(t)}\right]^{\kappa} - 1\right\} = \frac{1}{\rho} p_{FK}(z,t). \qquad (14)$$

Froude-Krylov pressure

According to the linear theory of the water surface waves, Froude-Krylov pressure is related to wave amplitude a_W and frequency ω, frequency of encounter ω_e and time t by the following relation (Newman, 1984)

$$p(t) = \rho\, g\, a_W \exp\left(-\omega^2 \zeta / g\right) \sin\left(\omega_e t + \alpha\right), \qquad (15)$$

where ζ is depth at the moonpool bottom, frequency of encounter is

$$\omega_e = \omega - \frac{\omega_W^2}{g} V_S \cos\beta \qquad (16)$$

and V_S and β are ship speed and heading respectively ($\beta = 180^0$ for the head seas).

In this analysis depth ζ is dependent upon the heaving motion of ship at the location of a moonpool as follows

$$\zeta = T - z_3. \tag{17}$$

Method validation

The method was validated by conducting model tests in the towing tank of the Ship Laboratory of the Helsinki University of Technology. A simple uncovered stationary cylindrical model of a moonpool was used in the tests. The draft of the model was 0.3 m and radius R = 0.11 m. Tests were conducted in regular (sinusoidal form) waves. The oncoming wave and motion of the water column motion in the model were measured. The results of the tests are shown in figure 3.

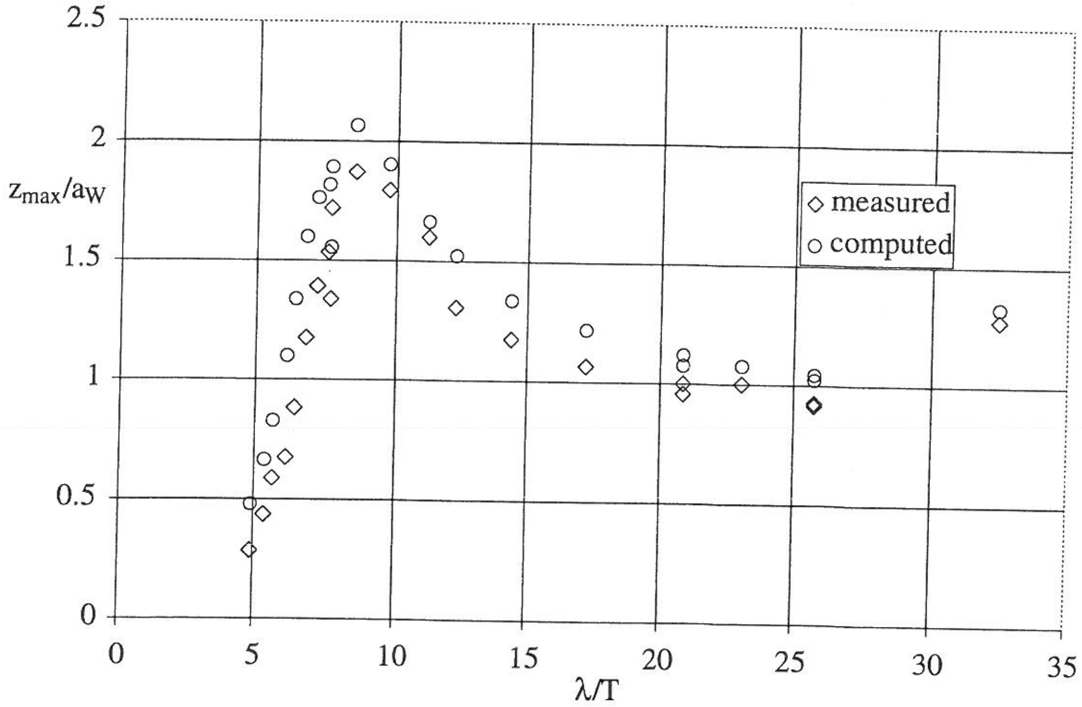

Fig. 3 Computed and measured in model scale water column maximum motion compared to regular wave amplitude as a function of wave length to draft ratio. Moonpool is stationary.

In the same figure the results of the numerical simulations are presented, too. Simulations were conducted by a numerical integration of the equation of motion (13) using Runge-Kutta algorithm. Zero initial conditions were applied, ie. $z = \dot{z} = 0$ for $t = 0$. Same regular waves, that were measured in the tests, were used also in the numerical simulations. Simulation was run for a sufficiently long time in order to achieve a steady state condition of the response. Because of the non-linearities of the phenomenon, the peak values of the responses related to the wave amplitude are presented instead of the ratio of amplitudes of the first harmonics.

There is a general trend of the numerical results to overestimate the responses. The relative difference is highest in the short wave region. Measured and numerically evaluated peak responses occur at the wave length to draft ratio $\lambda/T \approx 8$. This agrees well with the critical value deduced from the natural frequency of the linearized version of equation (13). The homogeneous, linearized version of this equation is

$$\ddot{z}\left(T + \frac{2}{3} R\, C_A\right) + g\, z = 0. \tag{18}$$

Faltinsen (1990) has also presented a linearized version of the water column motion in a moonpool. He, however, disregards the effect of the added mass. Taking for the added mass coefficient $C_A = 1$, yields natural frequency of the linearized model

$$\omega_0 = \sqrt{g/\left(T + \frac{2}{3} R\, C_A\right)} = 5.13 \text{ rad/s}. \tag{19}$$

The critical wave length that is corresponding to the natural frequency can be evaluated from the formula

$$\lambda = 2\,\pi\, g/\omega_0^2 = 2.34 \text{ m} \tag{20}$$

that stems from the linear theory of surface waves in deep water (Newman, 1984). Thus the critical wave length to draft ratio is $\lambda/T = 7.8$. This value is close to the observed and simulated critical value.

An example of the method application

The method is applied for evaluating the motion of the water column in the moonpool of a research vessel operating in irregular head waves ($\beta = 180^{\circ}$). Ship length at the waterline is $L_W = 100$ m. Draft is 7 m. The height to the main deck H = 13 m. The side of a square shape cross section of the moonpool is b = l = 5 m. Vessel speed is 10 knots. Irregular waves are modelled by the Jonswap spectrum of the form (Faltinsen, 1990)

$$S(\omega) = 155\, \frac{H_{1/3}^2}{T_1^4\, \omega^5} \exp\left(\frac{-944}{T_1^4\, \omega^4}\right) (3.3)^{Y}, \tag{21}$$

where

$$\gamma = \exp\left[-\left(\frac{0.191\ \omega\ T_1 - 1}{\sqrt{2}\ \sigma}\right)^2\right] \tag{22}$$

and

$$\begin{aligned} \sigma &= 0.07 \ \text{ for } \omega \le 5.24/T_1 \\ \sigma &= 0.09 \ \text{ for } \omega > 5.24/T_1. \end{aligned} \tag{23}$$

T_1 is a mean wave period that is related to modal period T_0 by the expression $T_1 = 0.834\ T_0$. $H_{1/3}$ is the significant wave height. It is the mean value of the one third of the highest waves.

In the numerical simulations irregular waves are generated by dividing the continuous wave power spectral density into 100 regular waves. Discrete frequency values are generated by a random number generator covering the frequency range $0.25 < \omega < 2$ [rad/s]. For each spectral line wave amplitude is evaluated with a formula

$$a_{W,i} = \sqrt{2\ S(\omega_i)\ \Delta\omega_i}\,. \tag{24}$$

In the example simulations, irregular waves of the significant height $H_{1/3} = 6$ m and mean period of $T_1 = 8.2$ s are used. The wave spectrum is shown in figure 4.

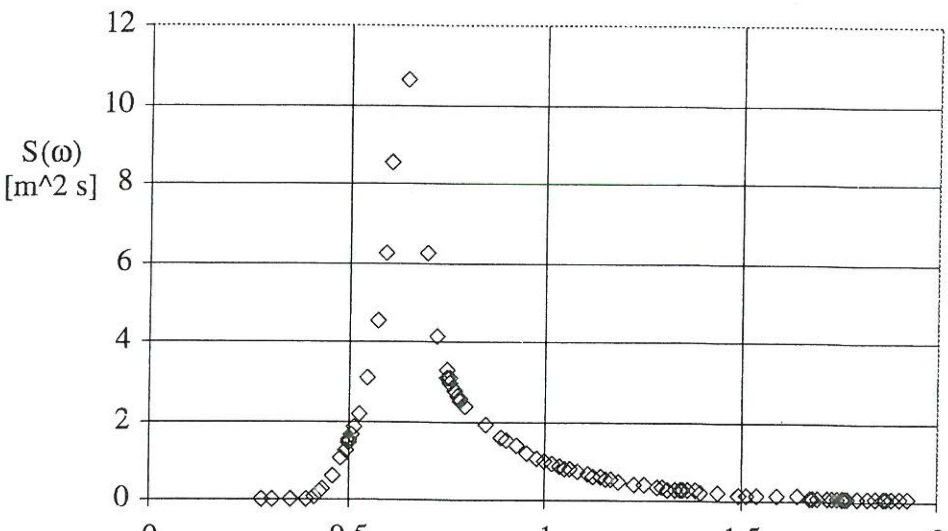

Fig. 4 Jonswap wave spectrum. Significant wave height $H_{1/3} = 6$ m. Mean period $T_1 = 8.2$ s.

Each wave component has random phase α_i generated also by a random number generator. As a result, the Froude-Krylov pressure corresponding to an irregular wave train is generated

$$p_{FK}(t) = \rho\, g \sum_{i=1}^{100} a_{W,i} \exp\left[-\omega_i^2 \left(T - z_3\right)/g\right] \sin\left(\omega_{e,i}\, t + \alpha_i\right). \tag{25}$$

Integration of equation (13) is conducted with a time step of 100 ms for half an hour of ship operation. Added mass coefficient is taken to be $C_A = 1$.

Ship motions are normally regarded to be linear in respect to the amplitudes or slopes of the encountered waves. Motion linearity makes it possible to evaluate the heaving motion at the location of the moonpool as a superposition of the motion harmonics. As a result of this assumption, heave motion z_3 is prescribed by the sum

$$z_3(t) = \sum_{i=1}^{100} a_{W,i} \left|H(\omega_{e,i})\right| \sin\left(\omega_{e,i}\, t + \beta_i\right), \tag{26}$$

where $\left|H(\omega_{e,i})\right|$ and β_i are the gain factor and phase of the transfer function that describes the heave motion of the ship at the location of the moonpool. Transfer function of ship motions are evaluated by a standard method implementing a linear strip theory (Kaplan & Raff, 1972). Gain factor and phase of the heaving motion at the moonpool location of the example ship are shown in figures 5 and 6 respectively.

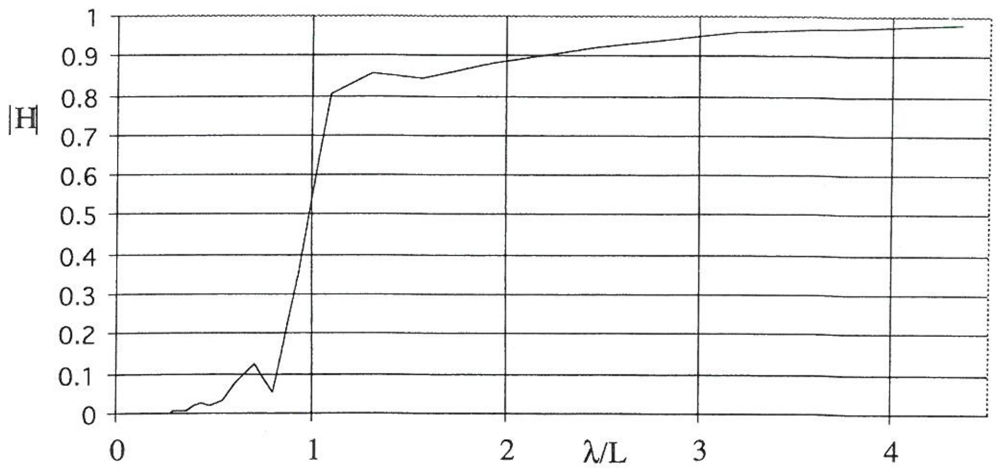

Fig. 5 Gain factor of the heaving motion at the moonpool location as a function of wave length in relation to ship length.

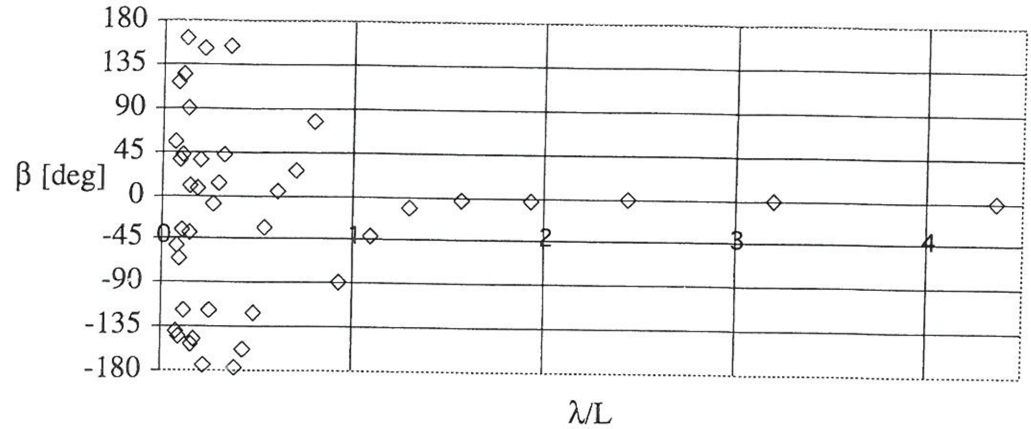

Fig. 6 Phase of the heaving motion with respect to wave elevation at the moonpool location as a function of wave length in relation to ship length.

An example of the evaluated water column motion z, wave a_W and heaving motion z_3 is presented in figure 7 for an open moonpool.

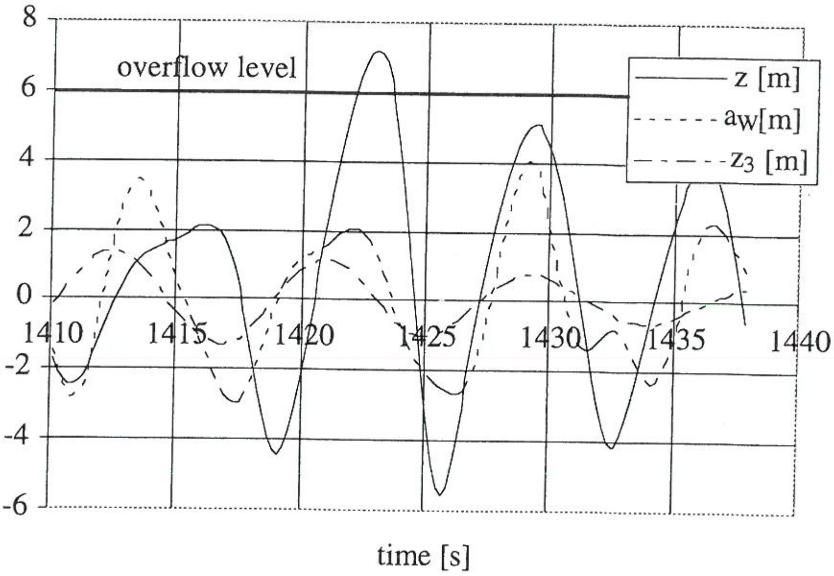

Fig. 7 An example of the evaluated water column motion z, wave a_W and heaving motions z_3.

In this example an event of water overtopping well edge is visible. In order to evaluate the amount of water that passes on the deck during one hour of ship operation overflow flux is evaluated by the formula

$$Q\ [m^3/m^2/hour] = \frac{1\ hour}{0.5\ hour} \int_0^{0.5\ hour} \dot{z}(t)\,dt \quad \text{for } z(t) \geq H. \tag{27}$$

Thus Q represents the volume of water overtopping well edge per well area A [m^2]. In the considered case the volume of water overtopping the well edge is $Q = 11\ m^3/m^2/hour$.

Because of the non-linear nature of water column motion, the following quantities are moreover evaluated:

- Maximum z_{max} [m] and minimum z_{min} [m] values of water column motion
- Significant values of maxima $z_{max,1/3}$ [m] and minima $z_{min,1/3}$ [m]
- Zero-crossing period T_Z [s] of water column motion

In the considered case these values are:

- $z_{max} = 7.2$ m, $z_{min} = -5.9$ m
- $z_{max,1/3} = 5.5$ m, $z_{min,1/3} = -4.6$ m
- $T_Z = 6.4$ s.

An example of the evaluated water column motion z, wave a_W, heaving motion z_3 and dynamic air-pressure p_e is presented in figure 8 for the hermetically covered moonpool.

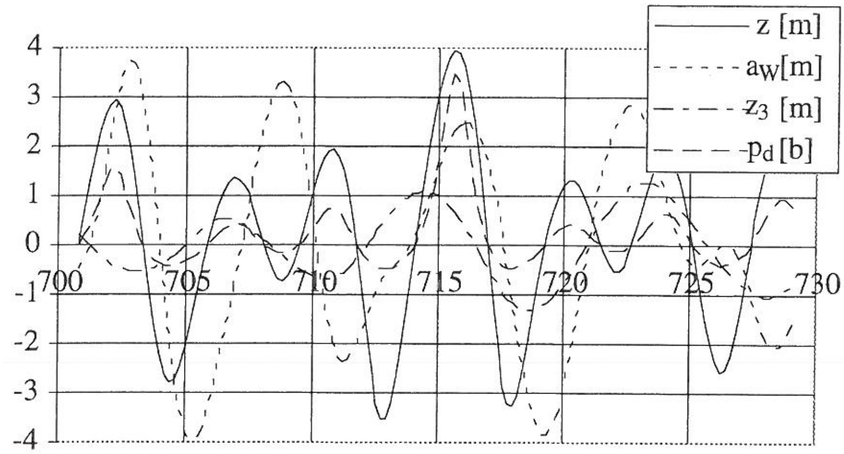

Fig. 8 An example of the evaluated water column motion z, wave a_W, heaving motion z_3 and dynamic air-pressure p_e for the hermetically covered moonpool.

The water column motion is smaller in this case. Dynamic air pressure that is given by formula

$$p_d = p_a \left\{ \left[\frac{H - T}{H - T - z(t)} \right]^{\kappa} - 1 \right\} \tag{28}$$

increases the frequency of water column motion. In the considered case the zero-crossing period of this motion is $T_z = 4.7$ s. Water column motion is almost symmetrical with respect to the still water level. Maximum and minimum values are

z_{max} = 3.9 m and z_{min} = − 3.9 m. The corresponding significant values are $z_{max,1/3}$ = 2.9 m and $z_{min,1/3}$ = − 2.9 m. The maximum value of the dynamic pressure is p_e = 3.5 b.

Conclusions

The non-linear effects are important when investigating water column motion in a moonpool. The main non-linearity stems from the fact that these large amplitude motions are associated with a substantial variations of an oscillating mass of water. The reaction forces acting at ship bottom are also of a non-linear nature. This precludes the use of frequency domain analysis that is common for evaluation of ship motions. Time domain analysis should be used instead. The critical frequency of this motion can be estimated with an aid of a linearized model. This frequency is mainly dependent on the draft of ship. Large amplitude motions of the water column are likely to occur if critical frequency is in the range of wave excitation. With an insufficient clearance of still water level and the moonpool top, a dangerous flooding of water on the deck may easily occur. Covering a moonpool by a hermetic hatch solves this problem. However a substantial air pressure load has to be taken into account when designing the cover.

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