ACCURATE AND APPROXIMATE ANALYSIS OF STATICAL BEHAVIOUR OF SUSPENSION BRIDGES

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SUMMARY: Behaviour of the girder-stiffened suspension bridges with geometrically nonlinear parabolic cables will be analysed in this paper. The common assumptions about the linear elastic strain-stress dependence of the material, absence of elongations and horizontal displacements of hangers and balancing of the initial vertical load by the cables only, have been taken into account. Also numerical examples are presented.

INTRODUCTION

Behaviour of the girder-stiffened suspension bridges with geometrically nonlinear parabolic cables will be analysed in this paper. The common assumptions about the linear elastic strain-stress dependence of the material, absence of elongations and horizontal displacements of hangers and balancing of the initial vertical load by the cables only, have been taken into account.

For the initial state of equilibrium of the cable we have

$$H_o \frac{d^2 z}{dx^2} = p_o \tag{1}$$

where p_o is the initial vertical load, H_o is the inner force horizontal component of the cable, and z and x are the initial coordinates of the cable.

By the action of temporary loads we may write for the cable (Figure 1)

$$H(\frac{d^2 z}{dx^2} + \frac{d^2 w}{dx^2}) = p_o + p' \tag{2}$$

and for the stiffening girder

$$-EI \frac{d^4 w}{dx^4} = p'' \tag{3}$$

where p' is the part of the additional load, balanced by the cable, p'' is the rest of additional load, balanced by the stiffening girder, H is the summary inner force horizontal component of the cable, $w=w(x)$ are the vertical displacements of the cable and the stiffening girder equivalently, and EI is the bending rigidity of the stiffening girder.

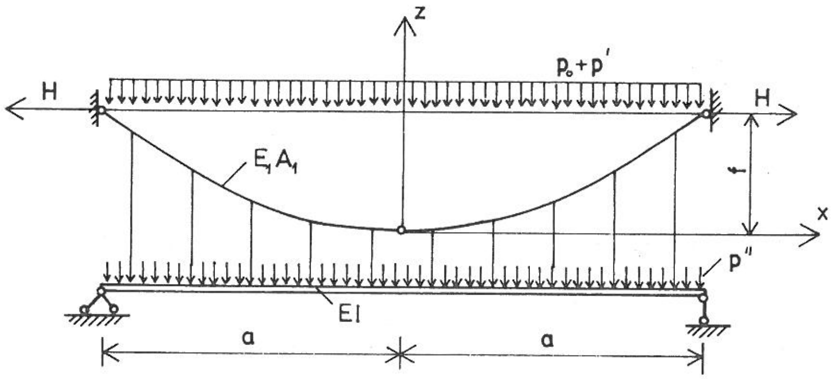

Figure 1. Suspension bridge with nonlinear parabolic cable.

By summing the equations (2) and (3) we may present the condition of equilibrium in the following form

$$EI\,\frac{d^4w}{dx^4} - H\left(\frac{d^2z}{dx^2} + \frac{d^2w}{dx^2}\right) + p_o + p' + p'' = 0\,. \tag{4}$$

If we let p denote the sum of the dead and the whole temporary load and $c^2 = \frac{EI}{H}$, we may write

$$\frac{d^4w}{dx^4} - \frac{1}{c^2}\frac{d^2w}{dx^2} = \frac{1}{c^2}\frac{d^2z}{dx^2} - \frac{p}{EI}\,. \tag{5}$$

To solve the differential equation (5) by the given initial cable form and boundary conditions we have to determine the value of the coefficient c. This value is related to the cable deformations and is to be found from the compatibility condition of its relative elongations. This condition may be written as the equation of equality of geometrical and elastic elongations:

$$\frac{1}{1+\left(\frac{dz}{dx}\right)^2}\left[\frac{du}{dx} + \frac{dw}{dx}\left(\frac{dz}{dx} + \frac{1}{2}\frac{dw}{dx}\right)\right] = \frac{H-H_o}{E_1A_1}\left[1+\left(\frac{dz}{dx}\right)^2\right]^{1/2} \tag{6}$$

where u is the horizontal displacement of the cable and E_1A_1 is the rigidity in tension of the cable.

After integrating the equation (6) for symmetrical loading over the half of the cable span we have

$$\int_0^a \frac{du}{dx}\,dx + \int_0^a \frac{dw}{dx}\left(\frac{dz}{dx} + \frac{1}{2}\frac{du}{dx}\right)dx = \frac{H-H_o}{E_1A_1}\int_0^a \left[1+\left(\frac{dz}{dx}\right)^2\right]^{3/2} dx \;. \tag{7}$$

The first member of the equation (7) represents the horizontal displacement of the supporting point of the cable. On the other hand, the displacement concerned may be expressed as the product of the increment of the cable force and the shift of the support under the action of the unit force. In the case of vertical pylons and the inclined anchor cables (Figure 2) we have

$$\int_0^a \frac{du}{dx}\,dx = \frac{(H-H_o)\,b}{E_2A_2\cos^2\beta} \tag{8}$$

where b is horizontal projection of the anchor cable length, β is the inclination angle of the anchor cable and E_2A_2 is the tension rigidity of the anchor cable.

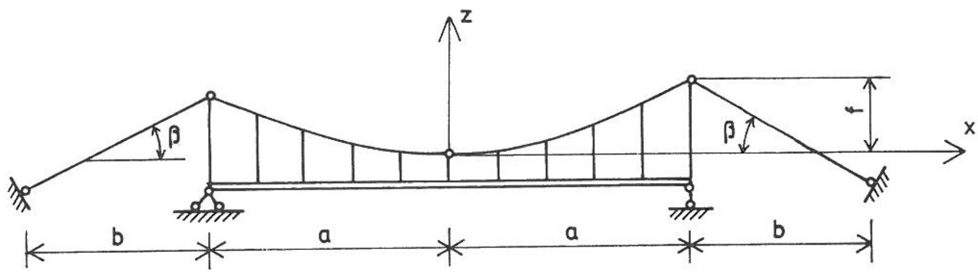

Figure 2. Suspension bridge with vertical pylons.

ACTION OF THE UNIFORMLY DISTRIBUTED LOAD

For the parabolic cable the co-ordinates x and z are connected by the quadratic equation

$$z = fx^2/a^2 \;. \tag{9}$$

Taking into account the boundary conditions

$$w = 0 \quad \text{and} \quad \frac{d^2w}{dx^2} = 0 \quad \text{by } x = \pm a$$

$$\frac{dw}{dx} = 0 \quad \text{and} \quad \frac{d^3w}{dx^3} = 0 \quad \text{by } x = 0$$

we may write the solution of the equation (5) in the form

$$\frac{w}{f} = (\frac{pa^2c^2}{2EIf} - 1)\left[2\,\frac{c^2}{a^2}\,(1-\frac{\mathrm{ch}x/c}{\mathrm{ch}a/c}) - (1-x^2/a^2)\right]. \qquad (10)$$

The corresponding derivatives

$$\frac{dw}{dx} = (\frac{pac^2}{EI} - 1)(\frac{x}{a} - \frac{c}{a}\,\frac{\mathrm{sh}x/c}{\mathrm{ch}a/c}) \qquad (11)$$

$$\frac{d^2w}{dx^2} = (\frac{pc^2}{EI} - \frac{2f}{a^2})(1 - \frac{\mathrm{ch}x/c}{\mathrm{ch}a/c}). \qquad (12)$$

To integrate the right-hand member of equation (7) it is useful to develop the expression in the square brackets as a series in the powers of the function

$$(\frac{dz}{dx})^2 = \frac{4f^2x^2}{a^4}.$$

So we may write

$$\int_o^a \left[1 + (\frac{dz}{dx})^2\right]^{3/2} dx = a(1 + 2f^2/a^2 + 6f^4/5a^4 - 4f^6/7a^6 + \ldots). \qquad (13)$$

In the real conditions $f/a \leq 1/4$ and we may neglect the third and the next members of the series (13) as small values in comparison with the unity and the right side of the equation (7) takes the form

$$\frac{H-H_o}{E_1A_1}\int_o^a \left[1+(\frac{dz}{dx})^2\right]^{3/2} dx = \frac{(H-H_o)a}{E_1A_1}(1 + 2f^2/a^2). \qquad (14)$$

Taking into account (9) and (11) we get after integrating the second member of equation (7)

$$\int_o^a \frac{dw}{dx}\,(\frac{dz}{dx} + \frac{1}{2}\,\frac{dw}{dx})\,dx = \frac{4f^2}{a^2}\,(-\frac{a^3}{6} - \frac{ac^2}{4} + \frac{ac^2}{4}\,\mathrm{th}^2\,\frac{a}{c} + \frac{c^3}{4}\,\mathrm{th}\,\frac{a}{c}) +$$
$$+ \frac{2pfc^2}{EIa^2}\,(\frac{a^3}{6} + \frac{ac^2}{4} - \frac{ac^2}{4}\,\mathrm{th}^2\,\frac{a}{c} - \frac{c^3}{4}\,\mathrm{th}\,\frac{a}{c}) + \qquad (15)$$
$$+ (\frac{pc^2}{EI})^2(\frac{a^3}{6} - \frac{5ac^2}{4} + \frac{ac^2}{4}\,\mathrm{th}^2\,\frac{a}{c} + \frac{5c^3}{4}\,\mathrm{th}\,\frac{a}{c}).$$

Now the equation (7) may be written as follows

$$\frac{2}{3} + \frac{c^2}{a^2}(1-\mathrm{th}^2\,\frac{a}{c}) - \frac{c^3}{a^3}\,\mathrm{th}\,\frac{a}{c} + \frac{EI(1+\psi)}{2E_1A_1f^2}\,\frac{a^2}{c^2} - \frac{p_oa^4(1+\psi)}{2E_1A_1f^3} =$$

$$= \frac{pa^4}{2EIf}\frac{c^2}{a^2}\left[4\frac{c^2}{a^2} + 2\frac{c^2}{a^2}(1-\text{th}^2\frac{a}{c}) - 6\frac{c^3}{a^3}\text{th}\frac{a}{c}\right] + \tag{16}$$

$$+ \left(\frac{pa^4}{2EIf}\right)^2\frac{c^4}{a^4}\left[\frac{2}{3} - 4\frac{c^2}{a^2} - \frac{c^2}{a^2}(1-\text{th}^2\frac{a}{c}) + 5\frac{c^3}{a^3}\text{th}\frac{a}{c}\right]$$

where $\psi = 2f^2/a^2 + \dfrac{bE_1A_1}{aE_2A_2\cos^2\beta}$.

The solution of the problem involves the determination of the coefficient c from the transcendental equation (16) and computation of the inner force horizontal component $H = c^2EI$ and the vertical displacements parameter w_o

$$\frac{w_o}{f} = \left(\frac{pa^2c^2}{2EIf} - 1\right)\left[2\frac{c^2}{a^2}\left(1-\frac{1}{\text{ch}\frac{a}{c}}\right) - 1\right] . \tag{17}$$

The bending moment of the stiffening girder is expressed as

$$M = -EI\frac{d^2w}{dx^2} = \left(\frac{2EIf}{a^2} - pc^2\right)\left(1 - \frac{\text{ch}x/c}{\text{ch}a/x}\right) . \tag{18}$$

For the approximate solution of the problem it is useful to prescribe the displacements function in the form

$$w = -w_o\cos\frac{\pi x}{2a} \tag{19}$$

which satisfies the boundary conditions.

The compatibility condition of the cable elongations (7) may be written now in the form

$$H - H_o = \frac{\pi^2E_1A_1f^2}{16a^2(1+\psi)}\left(\frac{w_o^2}{f^2} + \frac{64}{\pi^3}\frac{w_o}{f}\right) . \tag{20}$$

From the conditions of equilibrium (1)...(3) we have

$$(H - H_o)\frac{d^2z}{dx^2} + H\frac{d^2w}{dx^2} - EI\frac{d^4w}{dx^4} = p_1 \tag{21}$$

where $p_1 = p' + p''$ is total vertical temporary load.

Taking into account (19), (20) and (21) we may write

$$\frac{w_o^3}{f^3}\cos\frac{\pi x}{2a} + \frac{w_o^2}{f^2}\left(\frac{64}{\pi^2}\cos\frac{\pi x}{2a} + \frac{8}{\pi^2}\right) + \frac{w_o}{f}\left[\frac{512}{\pi^5} + \frac{4EI(1+\psi)}{E_1A_1f^2}\cos\frac{\pi x}{2a} + \right.$$

$$+ \frac{8p_o a^4(1+\psi)}{\pi^2 E_1 A_1 f^3} \cos\frac{\pi x}{2a} \Big] = \frac{64 p_1 a^4 (1+\psi)}{\pi^4 E_1 A_1 f^3} . \tag{22}$$

Using the Bubnov-Galerkin procedure we get the equation for determination of the displacement parameter in the form

$$\frac{w_o^3}{f^3} + \frac{96}{\pi^3}\frac{w_o^2}{f^2} + \left[\frac{2048}{\pi^6} + \frac{4EI(1+\psi)}{E_1 A_1 f^2} + \frac{8p_o a^4(1+\psi)}{\pi^2 E_1 A_1 f^3}\right]\frac{w_o}{f} = \frac{256 p_1 a^4(1+\psi)}{\pi^3 E_1 A_1 f^3} . \tag{23}$$

Now the horizontal component H of the cable force may be computed from the condition of compatibility (20)

$$H = H_o + \frac{4E_1 A_1 f^2}{\pi a^2 (1+\psi)} \frac{w_o}{f} \left(1 + \frac{\pi^3}{64}\frac{w_o}{f}\right) . \tag{24}$$

The bending moment of the stiffening girder

$$M = -EI\frac{d^2 w}{dx^2} = \frac{\pi^2 EIf}{4a^2}\frac{w_o}{f}\cos\frac{\pi x}{2a} . \tag{25}$$

LOADING THE SUSPENSION BRIDGE BY UNIFORMLY DISTRIBUTED LOADS ON THE WHOLE AND ON HALF SPAN

Let us denote p_o the initial vertical load, balanced by the cable; $p=p_o+p_1$ the sum of initial and additional load, applied on the whole span; p_2 the additional load, applied on the right half of the bridge span.

In this case we have the condition of equilibrium in the following form: for the left half of the span $(x < 0)$

$$\frac{d^4 w}{dx^4} - \frac{1}{c^2}\frac{d^2 w}{dx^2} - \frac{2f}{a^2 c^2} + \frac{p}{EI} = 0 \tag{26}$$

and for the right half of the span $(x > 0)$

$$\frac{d^4 w}{dx^4} - \frac{1}{c^2}\frac{d^2 w}{dx^2} - \frac{2f}{a^2 c^2} + \frac{p+p_2}{EI} = 0 . \tag{27}$$

The equations (26) and (27) have to be solved under boundary conditions demanding absence of vertical displacements and bending moments at the supports and equality of displacement and its derivatives in the middle of the span.

So we get

by $x < 0$

$$\frac{w}{f} = \left(\frac{pa^2c^2}{2EIf} - 1\right)\left[2\frac{c^2}{a^2}\left(1 - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) - \left(1 - \frac{x^2}{a^2}\right)\right] +$$

$$+ \frac{p_2 a^2 c^2}{4EIf}\left[2\frac{c^2}{a^2}\left(ch\frac{a}{c} - 1\right)\left(\frac{sh\frac{x}{c}}{sh\frac{a}{c}} + \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) - \left(1 + \frac{x}{a}\right)\right] \tag{28}$$

by $x > 0$

$$\frac{w}{f} = \left[\frac{(p+p_2)a^2c^2}{2EIf} - 1\right]\left[2\frac{c^2}{a^2}\left(1 - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) - \left(1 - \frac{x^2}{a^2}\right)\right] +$$

$$+ \frac{p_2 a^2 c^2}{4EIf}\left[2\frac{c^2}{a^2}\left(ch\frac{a}{c} - 1\right)\left(\frac{sh\frac{x}{c}}{sh\frac{a}{c}} - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) + \left(1 - \frac{x}{a}\right)\right] . \tag{29}$$

For the middle of the span

$$\frac{w_o}{f} = \left[\frac{(p+0.5p_2)a^2c^2}{2EIf} - 1\right]\left[2\frac{c^2}{a^2}\left(1 - \frac{1}{ch\frac{a}{c}}\right) - 1\right] . \tag{30}$$

The bending moment of the stiffening girder

by $x < 0$

$$M = -EI\frac{dw}{dx} = \left(\frac{2EIf}{a^2} - pc^2\right)\left(1 - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) - \frac{p_2 c2}{2}\left(ch\frac{a}{c} - 1\right)\left(\frac{sh\frac{x}{c}}{sh\frac{a}{c}} + \frac{ch\frac{x}{a}}{ch\frac{a}{c}}\right) \tag{31}$$

by $x > 0$

$$M = \left(\frac{2EIf}{a^2} - pc^2\right)\left(1 - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) - \frac{p_2 c^2}{2}\left[2\left(1 - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right) + \left(ch\frac{a}{c} - 1\right)\left(\frac{sh\frac{x}{c}}{sh\frac{a}{c}} - \frac{ch\frac{x}{c}}{ch\frac{a}{c}}\right)\right] \tag{32}$$

Coefficient $c = \left(\frac{EI}{H}\right)^{1/2}$ is to be found from the compatibility condition of elongations, which may be presented in the following form

$$\frac{2}{3} + \frac{c^2}{a^2}\left(1 - th^2\frac{a}{c} - \frac{c^3}{a^3}\,th\frac{a}{c} + \frac{EI(1+\psi)}{E_1A_1f^2}\frac{a^2}{c^2} - \frac{p_o a^4(1+\psi)}{2E_1A_1f^3} =\right.$$

$$= \frac{(p+0.5p_2)a^2c^2}{2EIf}\left[4\frac{c^2}{a^2} + 2\frac{c^2}{a^2}\left(1 - th^2\frac{a}{c} - 6\frac{c^3}{a^3}\,th\frac{a}{c}\right] +\right.$$

$$+ \frac{p(p+p_2)a^4c^4}{(2EIf)^2}\left[\frac{2}{3} - 4\frac{c^2}{a^2} - \frac{c^2}{a^2}\left(1 - th^2\frac{a}{c} + 5\frac{c^3}{a^3}\,th\frac{a}{c}\right] +\right. \tag{33}$$

$$+ \left(\frac{p_2 a^2 c^2}{2EIf}\right)^2 \left\{ \frac{5}{24} - \frac{c^2}{a^2} \left[2 + \frac{1}{4} \left(1 - th^2 \frac{a}{c} + \frac{1}{2} \frac{ch\frac{a}{c} - 1}{sh\frac{a}{c}} \right] + \right.\right.$$

$$\left.\left. + \frac{5c^3}{4a^3} \left[th \frac{a}{c} + 2 \frac{ch\frac{a}{c} - 1}{sh\frac{a}{c}} \right]\right\}.$$

For the approximate calculation of displacements and inner forces of the bridge it is useful to distribute the load into symmetrical and antisymmetrical parts. If we denote as above p_o the initial load, p_1 the additional load over the whole span and p_2 the load on the right half of the span, the symmetrical part of the additional load will be $p_s = p_1 + 0.5p_2$ and the antisymmetrical part $p_t = 0.5p_2 \mathrm{sgn} x$.

To calculate the bridge under the action of the first part of the load formulas 23, 24 and 25 are to be used, changing the load p_1 to p_s. For the second part of load p_t we may approximate the vertical displacements by function

$$w = -w_1 \sin \frac{\pi x}{a} . \tag{34}$$

From the compatibility condition of the cable elongations we may write

$$H = H_1 + \frac{\pi^2 E_1 A_1 w_1^2}{4a^2 (1+\psi)} \tag{35}$$

where H_1 is the inner force horizontal component of the cable under the action of the loads p_o and p_s.

Now, using the condition of equilibrium, we get an equation for determination of the displacements parameter w_1 in the form

$$\frac{w_1^3}{f_1^3} + \left[\frac{4EI(1+\beta)}{E_1 A_1 f_1^2} + \frac{2(p_o+p_s)a^4(1+\psi)}{\pi^2 E_1 A_1 f_1^3}\right] \frac{w_1}{f_1} = \frac{16 p_t a^4 (1+\psi)}{\pi^5 E_1 A_1 f_1^3} \tag{36}$$

where in the value $f_1 = f + w_o$ the displacement under the action of the first part of the load p_s is to be taken into account.

The bending moment of the stiffening girder

$$M = - EI \frac{d^2 w}{dx^2} = \frac{\pi^2 EIf_1}{a^2} \frac{w_1}{f_1} \sin \frac{\pi x}{a} . \tag{37}$$

The summary displacements and inner forces of the bridge have to be found by summing their values, found for the corresponding parts of loads:

$$w = -w_o \cos \frac{\pi x}{2a} - w_1 \sin \frac{\pi x}{a} \tag{38}$$

$$H = \frac{p_o a^2}{2f} + \frac{4E_1A_1f^2}{\pi a^2(1+\psi)}\frac{w_o}{f}(1 + \frac{\pi^3}{64}\frac{w_o}{f}) + \frac{\pi^2E_1A_1f^2}{4a^2(1+\psi)}\frac{w_1^2}{f_1^2} \qquad (39)$$

$$M = \frac{\pi^2EIf}{4a^2}\frac{w_o}{f}\cos\frac{\pi x}{2a} + \frac{\pi^2EIf_1}{a^2}\frac{w_1}{f_1}\sin\frac{\pi x}{a}. \qquad (40)$$

NUMERICAL EXAMPLE

Let us have a suspension bridge in the form of pedestrian overpass with the following parameters:

1) the middle span $l = 2a = 45$ m ;
2) the sag of the cable $f = 6$ m ;
3) the side spans $b = 15$ m, $tg\beta = 0.4$;
4) the rigidity parameters of the stiffening girder (I 60 GOST 8239-72)
   $E = 0.21 \cdot 10^6$ MPa, $I = 75010$ cm^4 ;
5) the rigidity parameters of the cables (Ø 52 GOST 7676-73) $E_1 = E_2 = 0.16 \cdot 10^6$ MPa,
   $A_1 = A_2 = 19.6$ cm^2 ;
6) the loads of the bridge
   $p_o = 2.0$ kN/m - the initial load,
   $p_1 = 2.0$ kN/m - the dead load,
   $p_2 = 10.0$ kN/m - the live load.

The whole span under the action of maximal load

For the exact analysis we have the calculation parameters (by $p = p_o + p_1 + p_2 = 14.0$ kN/m);

$$\psi = 2f^2/a^2 + \frac{bE_1A_1}{aE_2A_2\cos^2\beta} = 0.9156$$

$$\frac{p_o a^4(1+\psi)}{2E_1A_1f^3} = 0.00725 ; \qquad \frac{EI(1+\psi)}{E_1A_1f^2} = 0.02673$$

$$\frac{pa^4}{2EIf} = 1.8982.$$

From the transcendental equation (16) we find $\frac{c}{a} = 0.7514$, $c = 16.91$ m and

$$H = \frac{EI}{c^2} = 551 \text{ kN}.$$

Now we get from (17)

$$\frac{w_o}{f} = 0.03073 \text{ and } w_o = 0.184 \text{ m.}$$

From (18) we find the maximal bending moment at $x = 0$

$$\max M = 136 \text{ kN/m}$$

and the corresponding stresses

$$\sigma = \frac{\max M \cdot h}{2I} = \frac{136 \cdot 10^3 \cdot 0.6}{2 \cdot 75010 \cdot 10^{-8}} = 54.4 \cdot 10^6 \text{Pa} = 54.4 \text{ MPa.}$$

By the approximate analysis the calculation parameters have the values

$$\frac{4EI(1+\psi)}{E_1A_1f^2} = 0.10692 \; ; \qquad \frac{8p_oa^4(1+\psi)}{\pi^2E_1A_1f^3} = 0.01175$$

$$\frac{256(p_1+p_2)a^4(1+\psi)}{\pi^5E_1A_1f^3} = 0.07276.$$

From the cubic equation (23) we have

$$\frac{w_o}{f} = 0.0310 \text{ and } w_o = 0.186 \text{ m.}$$

After that we get from (24)

$$H = \frac{p_oa^2}{2f} + \frac{4E_1A_1f^2}{\pi a^2(1+\psi)} \frac{w_o}{f}\left(1 + \frac{\pi^3}{64} \frac{w_o}{f}\right) = 84 + 466 = 550 \text{ kN.}$$

The maximal bending moment of the stiffening girder is

$$\max M = \frac{\pi^2EIf}{4a^2} \frac{w_o}{f} = 143 \text{ kNm.}$$

Thus, the approximate analysis gives the cable displacement and inner force, very close to the exact values, but the approximate bending moment differs from the exact value about 5 %.

The right half of the span under the action of live load

The parameters of the compatibility equation of the cable elongations (33) by $p = p_o + p_1 = 4.0$ kN/m and $p_2 = 10.0$ kN/m have the values

$$\frac{p_o a^4(1+\psi)}{2E_1A_1f^3} = 0.00725 \; ; \qquad \frac{EI(1+\psi)}{E_1A_1f^2} = 0.02673$$

$$\frac{pa^4}{2EIf} = 0.54234 \qquad ; \qquad \frac{p_2 d^4}{2EIf} = 1.3558 \, .$$

Solution of the transcendental equation (33) in this case gives

$$\frac{c}{a} = 0.9295, \quad \frac{a}{c} = 1.07585 \text{ and}$$

$$H = \frac{EI}{c^2} = \frac{EI}{a^2}\frac{a^2}{c^2} = 360 \text{ kN}.$$

From the equations (28) - (32) we find displacements and bending moments of the stiffening girder:

$w = 0.019$ m by $x = -0.5a$,

$w = 0.107$ m by $x = 0$,

$w = 0.171$ m by $x = +0.5a$,

$M = -220$ kN/m by $x = -0.5a$,

$M = +124$ kN/m by $x = 0$,

$M = +342$ kN/m by $x = +0.5a$

For the approximate solution of the problem we have to determine the parameters w_o and w_1 from (23) and (36), respectively, and to compute displacements and inner forces from (38), (39) and (40).

The first part of the load is characterized by the load parameter ($p_s = 2.0 + 0.5 \cdot 10.0 = 7.0$ kN/m)

$$\frac{256\, p_s a^4(1+\psi)}{\pi^5 E_1A_1f^3} = 0.042440$$

and from (23) we get

$$\frac{w_o}{f} = 0.01840, \quad w_o = 0.110 \text{ m}.$$

Now, for computing the displacements parameter of the secondary load we have

$$f_1 = f + w_o = 611 \text{ cm}.$$

The coefficients of the formula (36)

$$\frac{4EI(1+\psi)}{E_1A_1f^2} = 0.10310 \; ; \qquad \frac{2(p_o+p_s)a^4(1+\psi)}{\pi^2E_1A_1f^3} = 0.01252$$

$$\frac{16p_ta^4(1+\psi)}{\pi^5E_1A_1f^3} = 0.001794$$

and the unknown $\frac{w_1}{f} = 0.0155$; $w_1 = 0.095$ m.

Now we may compute by formulas (38) - (40):

the displacements

$w = 0.110 \cdot 0.7071 - 0.095 \cdot 1.0 = 0.017$ m by $x = -0.5a$

$w = 0.110 \cdot 1.0 = 0.110$ m by $x = 0$,

$w = 0.110 \cdot 0.7071 + 0.095 \cdot 1.0 = 0.172$ m by $x = 0.5a$;

the inner force horizontal component

$H = 367$ kN ;

the bending moments

$M = 4600 \cdot 0.01840 \cdot 0.7071 - 18760 \cdot 0.0155 \cdot 1.0 = -231$ kNm by $x = -0.5a$

$M = 4600 \cdot 0.01840 \cdot 1.0 = 85$ kNm by $x = 0$,

$M = 4600 \cdot 0.01840 \cdot 0.7071 + 18760 \cdot 0.0155 = 351$ kNm by $x = +0.5a$.

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