ON SOIL DEFORMATIONS

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PREFACE

One of the most important areas of research in the geotechnical field is that of developing equipment suitable for studying the deformative properties of soils. Equal importance can be attached to the development of methods for the estimation of settlement in structures founded direct upon the ground. Although noteworthy results have been achieved in this field of study, the development of methods that are sufficiently reliable and uncomplicated for determination of the deformative properties of soils has not as yet been successful. The accuracy of the methods of calculation available for estimation of the settlement in structures cannot be considered adequate under all the conditions that arise. Consequently, the methods applied for both investigation and calculation need to be improved, and their suitability should be checked "on a natural scale", by means of the long-term settlement observed in various structures.

STRESS-STRAIN FUNCTIONS

Ylinen [16], in his textbook on the theory of elasticity and strength of materials, observed that "to settle some problems that arise within the theory of the strength of materials, including that relating to the deflection and stress of a beam, and the buckling strength of struts, it is often advisable to approximate the compression test diagram of the material by means of an uncomplicated analytical expression". Ylinen termed this "uncomplicated analytical expression" a stress-strain function, and simultaneously emphasized that since the correlation between the stress and the compression is not known theoretically, only empirical expressions can be used for definition of the stress-strain function. Although serious attempts have also been made to develop stress-strain functions upon a theoretical and semi-theoretical basis, in all probability Ylinen's statement still holds good. Stress-strain functions are, and will continue to be "empirical expressions". However, stress-strain functions must be evolved with general constitutive theories as a starting-point, with care being taken that none of the theoretical and empirical information available in regard to the behaviour of building materials in different states of stress is omitted.

TYPES OF DEFORMATION

In materials that conform to the theory of linear elasticity, the generalized "Hooke's law" completely determines the correlation between stresses and strains. As a rule, the modulus of elasticity and Poisson's ratio are the constitutive parameters in Hooke's law.

Although soils do not keep to the theory of linear elasticity, the "principle" of Hooke's law can also be followed in geotechnics. In geotechnical applications, it is generally advisable to substitute the shear and the bulk modulus for the modulus of elasticity and Poisson's ratio respectively. On the introduction of these moduli, Hooke's law can be expressed by the equation (1)

$$\begin{aligned}
\varepsilon_1 &= \frac{\sigma_1 + \sigma_2 + \sigma_3}{9K} + \frac{\sigma_1 - \sigma_3}{6G} + \frac{\sigma_1 - \sigma_2}{6G} \\
\varepsilon_2 &= \frac{\sigma_1 + \sigma_2 + \sigma_3}{9K} + \frac{\sigma_2 - \sigma_1}{6G} + \frac{\sigma_2 - \sigma_3}{6G} \\
\varepsilon_3 &= \frac{\sigma_1 + \sigma_2 + \sigma_3}{9K} + \frac{\sigma_3 - \sigma_1}{6G} + \frac{\sigma_3 - \sigma_2}{6G}
\end{aligned} \tag{1}$$

$\varepsilon_1 \ldots \varepsilon_3$ are deformations (displacements) in the directions of principal stresses $\sigma_1 \ldots \sigma_3$

K is bulk modulus

G is shear modulus

According to Eq. (1), the displacement is the sum of volumetric strain and shear deformation. In building materials, which adhere to the theory of linear elasticity, it is assumed that no volumetric strains occur as the result of shear stresses. For this reason, Hooke's law does not include the displacement arising from volumetric strain, which is in turn induced by shear stresses. In soils, shear stresses produce not only shear deformations, but also volumetric strains. Knowledge of this type of deformation is inadequate as far as soil mechanics are concerned, although several studies have been made with a view to finding the factors that influence it. Apparently, the deformative properties of soils are more markedly dependent upon the "stress history" than are the deformations in other building materials. The influence exerted by time upon deformations must be taken into account in nearly all building materials; this time effect is a highly important factor, particularly in organic and cohesive soils.

CONSTITUTIVE VARIABLES

The choice of constitutive variables does not present any difficulties in study of the volumetric strains in soils. Table 1 lists the constitutive variables generally applied in this connection.

Table 1. Volumetric strain in soils. Constitutive variables.

Test	Constitutive variables		Notes
	Stress	Strain	
Oedometer test	σ_1	ε_1	$\varepsilon_2 = \varepsilon_3 = 0$
Compression test	p	ε_v	$\varepsilon_2 \neq 0;\ \varepsilon_3 \neq 0$

σ_1 is major principal stress

ε_1 is strain in direction of major principal stress

$p = 1/3\ (\sigma_1' + \sigma_2' + \sigma_3')$ mean value of principal stresses

$\varepsilon_v = \varepsilon_1 + \varepsilon_2 + \varepsilon_3$ volumetric strain

In the study of shear deformations, the task involved in selection of the constitutive variables is not so explicit in selection of the constitutive variables of functions that represent the volumetric strains. For this reason, use was made of a number of different variables in determination of the shear deformations [7]. Naturally, these variables are quite specifically inter-dependent.

Table 2 gives the constitutive variables employed below in study of the shear deformations of clay and sand.

Table 2. Shear strains. Constitutive variables

Constitutive variables		Soil
Stress	Strain	
σ_i and p	ε_i	Clay
q and p	ε_s	Sand

$$\sigma_i = \sqrt{\tfrac{1}{6}\,(\sigma_1' - \sigma_2')^2 + (\sigma_2' - \sigma_3')^2 + (\sigma_3' - \sigma_1')^2}$$

σ_i is intensity of shear stresses

$$\varepsilon_i = \sqrt{\tfrac{2}{3}\,(\varepsilon_1 - \varepsilon_2)^2 + (\varepsilon_2 - \varepsilon_3)^2 + (\varepsilon_3 - \varepsilon_1)^2}$$

ε_i is intensity of shear strains

$$q = \sigma_1' - \sigma_3'$$

$$\varepsilon_s = 2/3(\varepsilon_1 - \varepsilon_3)$$

$\sigma_1' \ldots \sigma_3'$ are effective principal stresses

As a rule Soviet scientists [15] [14] in particular use variables σ_i and ε_i. Variables q and ε_s have been applied to the "critical state" system (cf. [12]).

SOILS AND METHODS OF INVESTIGATION

The deformative properties of a typical cohesive and non-cohesive soil are treated below. The cohesive soil in this case is postglacial clay; the samples were taken from Salo. The cohesionless soil was test sand supplied by the Danish Geotechnical Institute

(G-12 sand). Fig. 1 illustrates the gradation curves for these samples.

Equipment manufactured by the Norwegian firm Geonor A/S was employed for the oedometer and triaxial tests on the clay sample. The triaxial tests were made in the form of undrained c-q-tests [7].

A triaxial apparatus, set up at the Danish Geotechnical Institute [3], was employed for the compression and shear tests on the sand sample. Moreover, the sand samples were studied in an oedometer constructed by the precision mechanical workshop of the Technical Research Centre of Finland [7].

VOLUMETRIC STRAIN

Fig. 2a presents the results obtained in oedometer tests on a sand sample. These were approximated by the stress-strain function (2)

$$\varepsilon_1 = a_o \sigma_1^{k_o} + C_o \qquad (2)$$

$$M = \frac{d\sigma_1}{d\varepsilon_1} = v_o\ {}_1^{\omega_o}$$

$\omega_o = 1 - k_o$

$v_o = \frac{1}{ak}$ $(k \neq 0)$

$\varepsilon_1 = \frac{\Delta h}{h_o}$, relative settlement of sample in oedometer test in direction of influence of stress σ_1

M is uniaxial tangent modulus

a_o, k_o, C_o, v_o are constants

Fig. 2a contains the plots calculated for stress-strain function (2). It is evident from the figure that the observations coin-

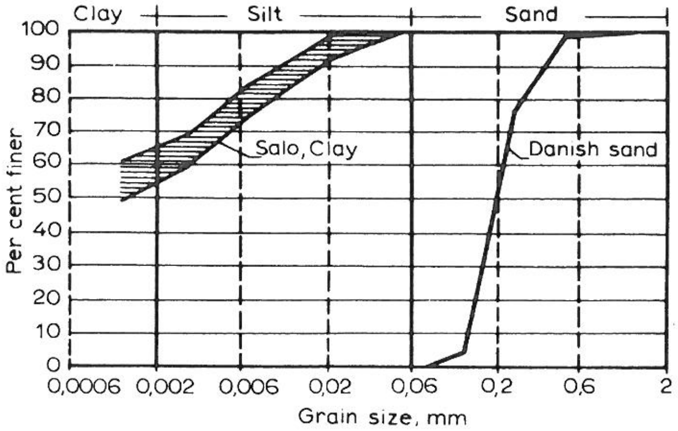

Fig. 1. Gradation of soils.

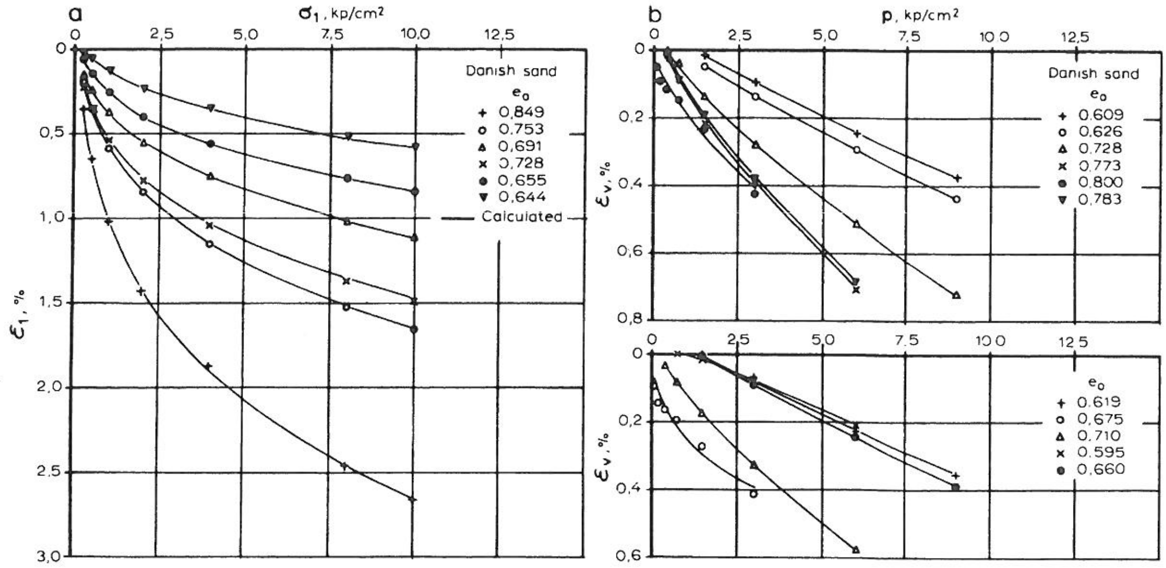

Fig. 2. Results obtained in oedometer and compression tests on sand sample.

cide well with the calculated curves. Furthermore, several researchers abroad, including the authors of [10], [11], [9], [8], [4], [5] and [6], have confirmed the suitability of Eq. (2) for the approximation of oedometer test results. Apparently Eq. (2) is applicable to all soils. According to the equation, the settlement will increase monotonously with increase in the stress. In reality, however, this is impossible, since in the oedometer test the sample is compacted (when breaking of the grains is prevented) at most by an amount corresponding to its initial porosity. Nevertheless, in practice this disadvantage is of little consequence. Difficulty would clearly be experienced in the formulation of a less complicated stress-strain function for this purpose.

Fig. 2b indicates the results obtained in compression tests made on the triaxial apparatus. The test results were approximated by means of Eq. (3) which is of the same form as Eq. (2).

$$\begin{aligned} \varepsilon_v &= a_c p^{k_c} + C_c \\ K &= \frac{dp}{d\varepsilon_v} = v_c p^{\omega_c} \\ \omega_c &= 1 - k_c \\ v_c &= \frac{1}{a_c k_c} \quad (k_c \neq 0) \end{aligned} \qquad (3)$$

$\varepsilon_v = \frac{\Delta V}{V_o}$ is the relative volumetric strain of the sample

K is bulk modulus (tangent modulus)

a_c, k_c, C_c, v_c are constants

Fig. 2b illustrates a plot of stress-strain functions (3), and confirms that in general the observations coincide to a reasonable degree of accuracy with the calculated theoretical plots. However, it needs to be observed that as a rule Eq. (2) is better suited for approximation of the oedometer test results than is Eq. (3) for approximation of the compression tests. For this reason, it may become necessary to substitute a "better and more natural" stress-strain function for Eq. (3).

The constitutive parameters of stress-strain functions (2) and (3) primarilly depend (within the same soil) upon the soil density. Fig. 3 indicates the relationship between exponent k in the above equations and the void ratio of the sample. It is discernible from the figure that k generally decreases with a decrease in the void ratio, and furthermore that $k_c \sim 3k_o$.

Fig. 4 indicates the relation between modulus number (v) and void ratio (e_o); in the present case, this can be approximated by Eq. (4)

$$v = \frac{1}{be_o + a} \qquad (4)$$

e_o is the initial void ratio of the sample

v is the modulus number (v_o or v_c)

a and b are constants ($a < 0$)

Eq. (4) can also be written in the following form (5)

$$e_o = \frac{n_o}{1 - n_o}$$

$$1 - n_o = \frac{bv}{1 + v(b - a)}$$

$$v = 0, \text{ when } n_o = 1 \qquad (5)$$

$$\frac{d(1 - n_o)}{dv} = b, \text{ when } n_o = 1$$

$$(1 - n_o) \rightarrow \frac{b}{b - a}, \text{ when } v \rightarrow \infty$$

n_o is initial porosity of the sample

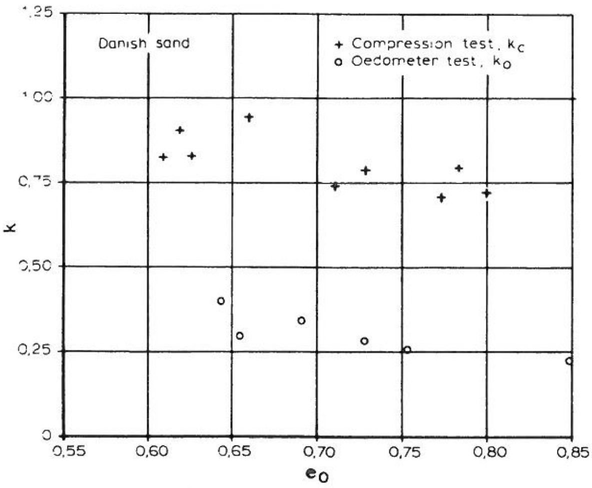

Fig. 3. Relation between void ratio and exponent k.

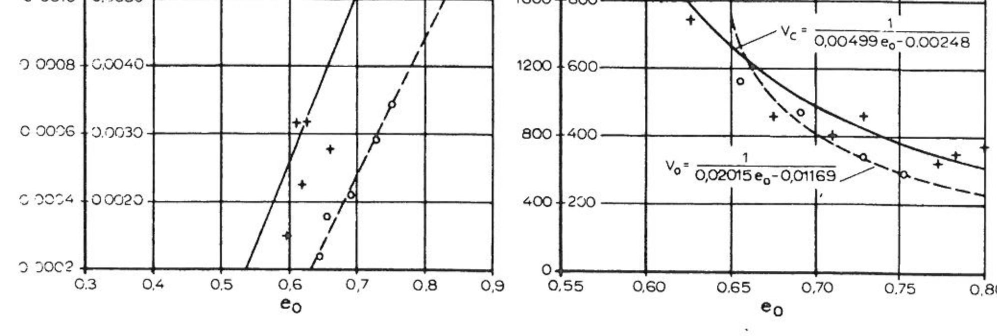

Fig. 4. Relation between void ratio and modulus number (v).

Fig. 5 tends to indicates that parameters a and b possess a specific physical meaning. When modulus number (v) approaches infinity, the porosity approaches a value -a/b-a, and the modulus number is zero, when n_o approaches the highest theoretical value, $n_o = 1.00$. In coarse-grained and tilly soils, parameters a and b are principally dependent upon the gradation and the grain form of the soil, whereas in fine-grained soils the parameters are dependent upon the plasticity properties (w_L and I_p).

SHEAR DEFORMATIONS

Fig. 6 illustrates the result of a shear test made on a clay sample. The "stress path" during shear is depicted in Fig. 7, and correspondingly the shear test results obtained with a sand sample are shown in Fig. 8.

It is observable from Figs. 6b and 8b that an almost linear correlation exists between the results observed in coordinate systems conforming to those of the figures. The stress-strain functions can consequently be expressed by Eqs. (6) and (7).

$$\sigma_i = \frac{\varepsilon_i}{1/G_o + 1/\sigma_{iu} \cdot \varepsilon_i} \qquad (6)$$

G_o is initial shear modulus (tangent modulus)

$\sigma_{iu} = \lim \sigma_i$, when $\varepsilon_i \to \infty$

$$q = \frac{\varepsilon_s}{1/E_o + 1/q_u \cdot \varepsilon_s} \qquad (7)$$

E_o is initial tangent modulus

$q_u = \lim q$, when $\varepsilon_s \to \infty$

The applicability of hyperbolic stress-strain functions (6) and (7)

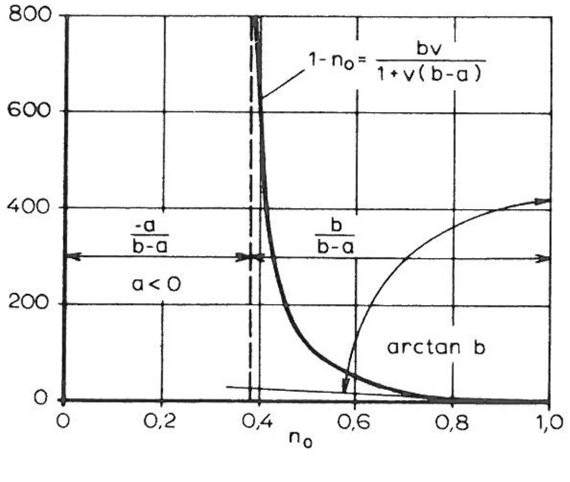

Fig. 5. Relation between porosity and modulus number.

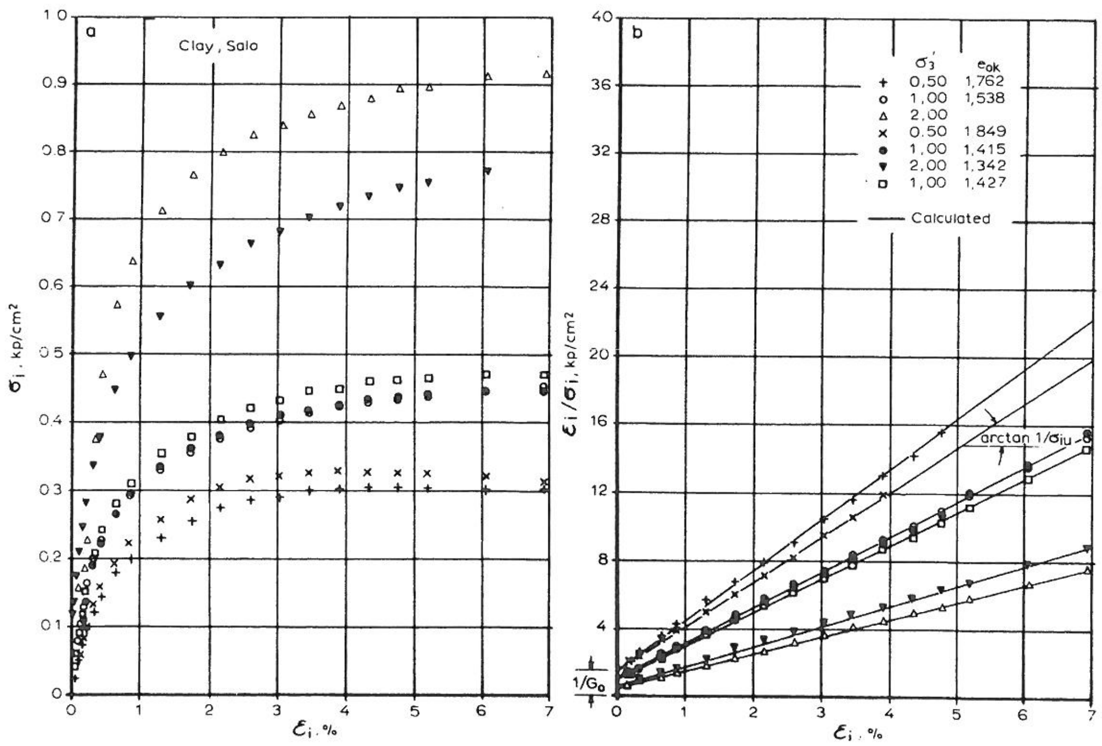

Fig. 6. Triaxial test result on clay sample.

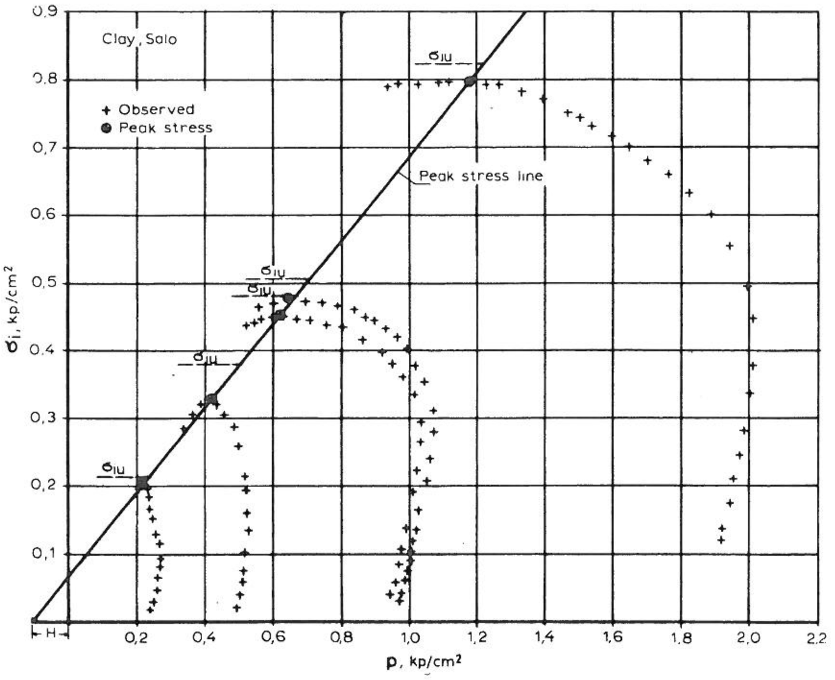

Fig. 7. Stress path in triaxial test on clay sample.

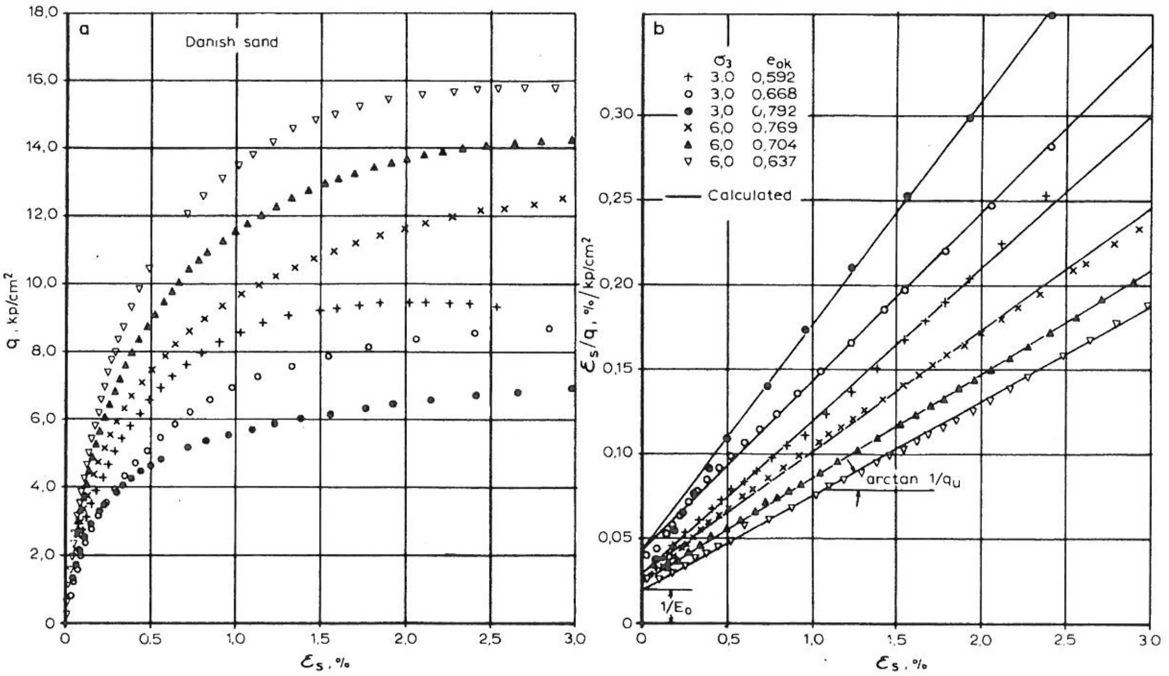

Fig. 8. Triaxial test result on sand sample.

for the approximation of shear deformations has been pointed out in a number of studies [7].

Fig. 9 illustrates the correlations between the constitutive parameters of Eqs. (6) and (7). The figures indicate that these correlations can be approximated by straight lines running through the origin of coordinates, i.e. the ratio of the initial tangent modulus to the theoretical strength (G_o/σ_{iu} and E_o/q_u) seems to be a constant that does not depend upon the void ratio or cell pressure (σ_3). If advantage is taken of this characteristic, functions (6) and (7) can be converted into invariant functions (8) and (9).

$$\frac{\sigma_i}{\sigma_{iu}} = \frac{\varepsilon_i}{\sigma_{iu}/G_o + \varepsilon_i} \qquad (8)$$

σ_i/G_o = 1/200 (Fig. 9a)

$$\frac{q}{q_u} = \frac{\varepsilon_s}{q_u/E_o + \varepsilon_s} \qquad (9)$$

q_u/E_o = 1/270 (Fig. 9b)

Stress-strain functions (8) and (9) have been plotted graphically in Figs. 10 and 11.

Fig. 12 depicts the relationship between the constitutive parameters of Eq. (7), viz. the void ratio and the cell pressure. The above relationships can be expressed by the following equations by application of the symbols of Fig. 12b.

$$E_o = \sigma_3^{k_c} \left[\frac{A_1}{e_{ok} - b_1} + a_1 \right] \qquad (10)$$

$$q_u = \sigma_3^{k_c} \left[\frac{A_2}{e_{ok} - b_2} + a_2 \right] \qquad (11)$$

σ_3 is effective cell pressure at the start of shear

e_{ok} is void ratio of the sample at the start of shear

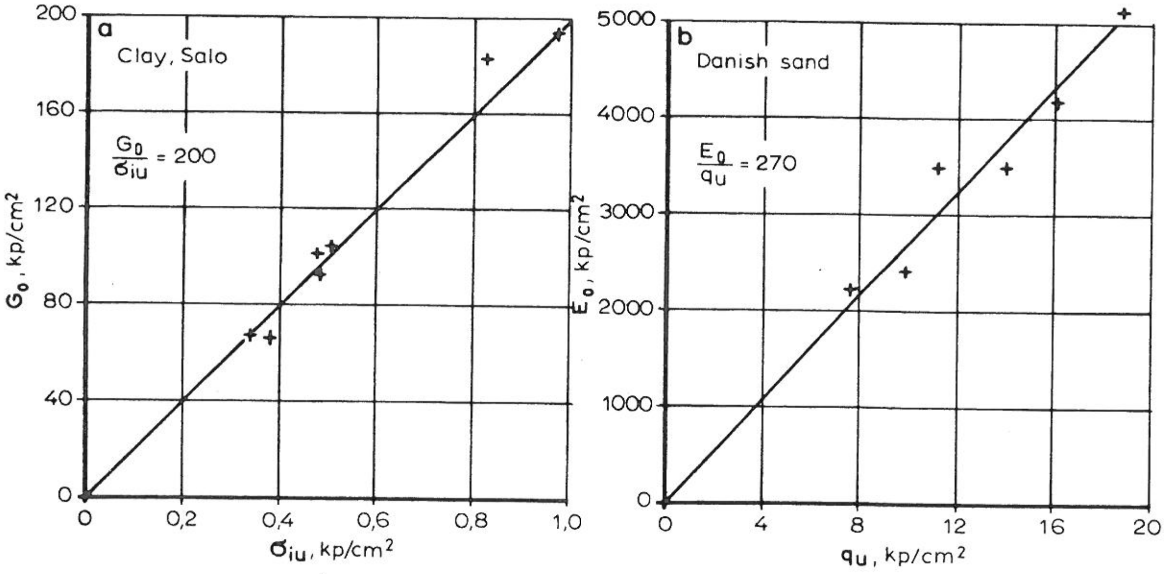

Fig. 9. Relation between parameters from stress-strain functions (6) and (7).

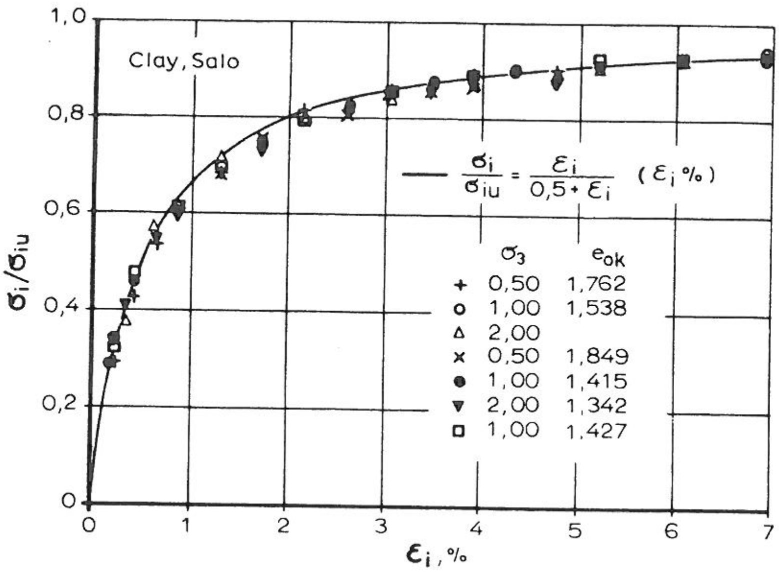

Fig. 10. Invariant deformation of clay sample.

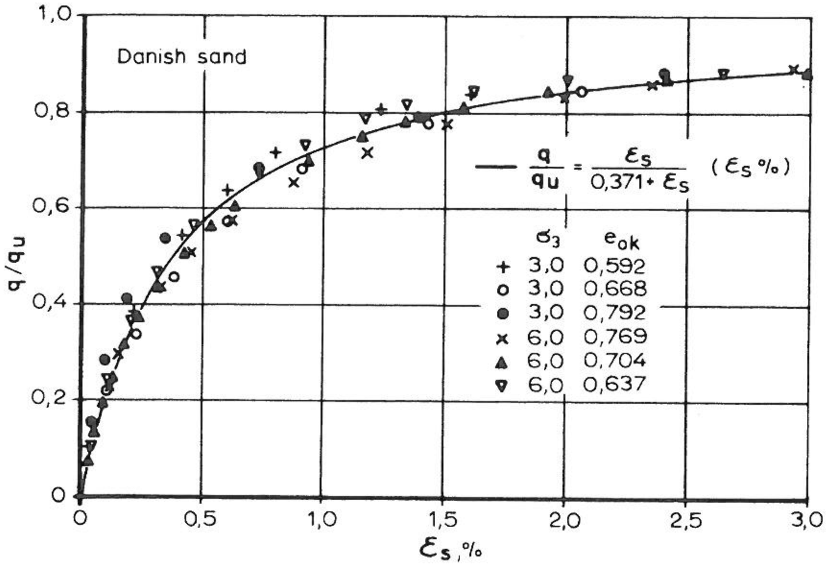

Fig. 11. Invariant deformation of sand sample.

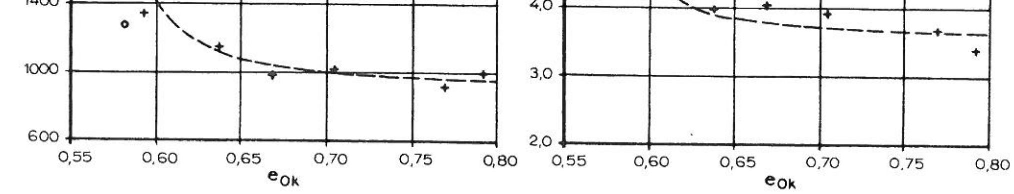

Fig. 12. Relation between void ratio and cell pressure, the parameters of Eq. (7).

k_c is an exponent from Eq. (3)

a, b and A are constants

It was observed above (Figs. 9 and 11) that E_o/q_u is obviously a constant that does not depend upon the cell pressure or the void ratio. Consequently, the ratio of the terms to be found within parentheses in Eqs. (10) and (11) represents the same constant.

Studies [2] and [1] have confirmed that the hyperbolic stress-strain function (6...9) is applicable for the calculation of shear strains in subsoil by the finite element method.

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