

SOME OBSERVATIONS CONCERNING THE RELATIONS BETWEEN  
POISSON'S RATIO AND YOUNG'S MODULUS OF CONCRETE

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1. AIMS OF THE INVESTIGATION

Commonly, the Poisson's ratio is ignored in conjunction with the dimensioning of concrete and steel structures. However, in those cases in which this ratio is included among the group of computational requirements, it often assumes the fixed value of  $1/6$ . In the literature, the only a small number of investigations have been published within the domain of elastic relations between the strain, measured in the plane orthogonal to the active force, and the strain confirmable in the direction of the force. Nonetheless, the steadily increasing number of high-class structures also implies exact knowledge of the properties of building materials; the calculations alone presuppose that the properties are known in all three directions. The main reason for this investigation was that of endeavouring to find a value for the Poisson's ratio, which would be suitable for computational use, and which would above all be represent the average value, and the one most expected in practice.

## 2. TEST SPECIMENS

In the study reported below, Poisson's ratio was determined by means of test specimens  $20 \times 20 \times 100$  cm in size. With the same concrete material, the cubic strength was determined by comparison test specimens,  $20 \times 20 \times 20$  cm in size. The test specimens are presented in Fig. 1.

The average 28-day cubic strength,  $K_{28}$ , was  $497 \text{ kp/cm}^2$  in test specimens denoted by strength class K 500, and  $269 \text{ kp/cm}^2$  in those of K 275. During the course of strength determination in the comparison test specimens, the loading was raised at a speed of  $5 \text{ kp/cm}^2/\text{sek}$ . In the prisms, the strength grew, as indicated in Fig. 2, as a function of the time.

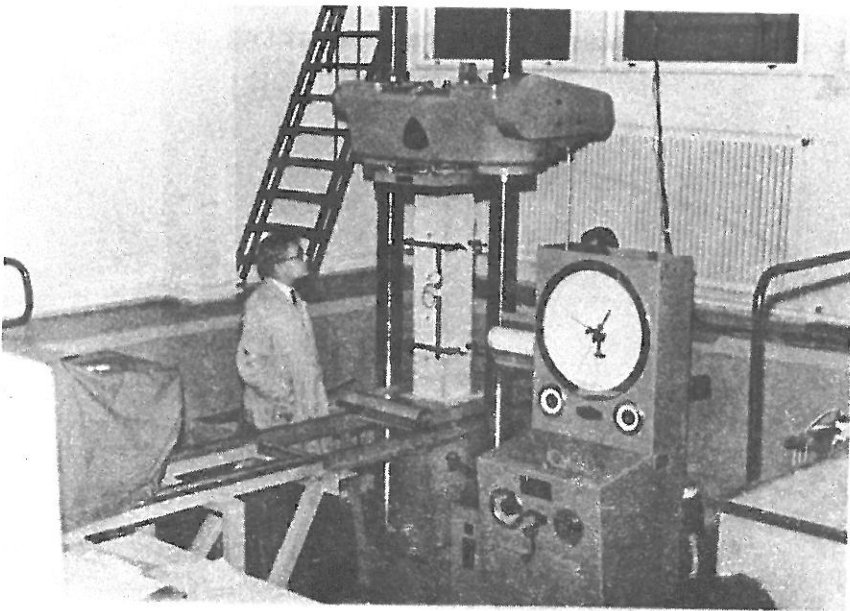


Fig. 1. Test specimen under load.

### 3. MAINTENANCE OF TEST SPECIMENS

The test prisms were first kept for 5 days in water, and subsequently under wet bags until they were aged 28 days. From this point on, they were kept at room temperature, with a relative air humidity of 40 %.

### 4. MEASURING INSTRUMENT

The strain was measured in the direction of the active force by the application of a deformer type Mays & Sohn, with 20 cm as the largest working base. The measurement studs were attached to the concrete surface by synthetic resin, soluble in acetone. The strain orthogonal to the direction of the force was measured by Strain-Gauge straps, 60 mm in length. The compression force was created by means of a Losenhausenwerk universal testing machine,

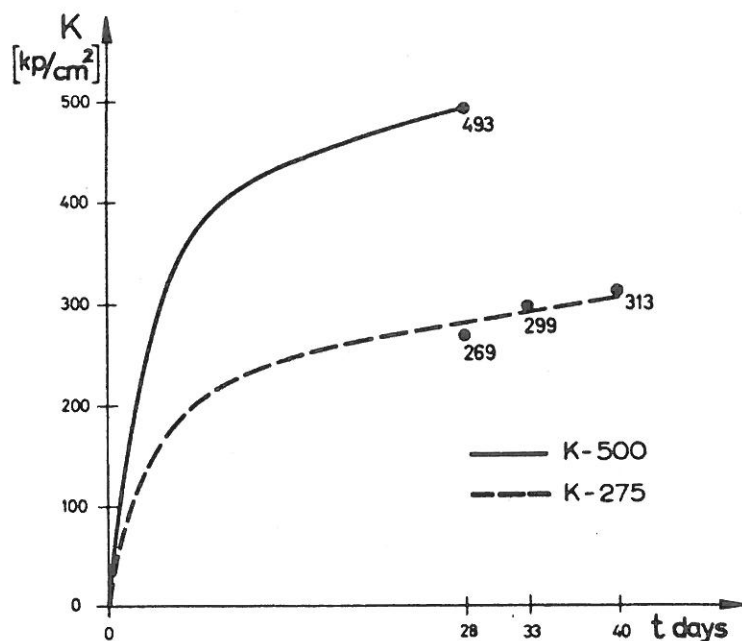


Fig. 2. Development of cubic strength of concrete specimens in strength classes K 500 and K 275 during the course of time.

which reliability tests had shown to attain an average working accuracy of 0.3 %.

## 5. YOUNG'S MODULUS IN RELATION TO THE AGE OF CONCRETE

In tests made in the K 500 strength class, it was confirmed that in concrete prisms, if the strength was increased in the manner indicated in Fig. 2, the strain corresponding to the same loading level remained almost constant. During the testing of this series, accordingly, no dependence of this kind was found in concrete between the strength and Young's modulus, which could be attributed to the development of cubic strength. Young's modulus retained a constant value, even if the cubic strength, or the strength as a whole, was increasing.

At the first loading time, the force acting upon the test specimens was raised step by step in the following manner:

$$0 - 0.1 K_{28}, \quad 0 - 0.2 K_{28}, \quad 0 - 0.3 K_{28}, \quad 0 - 0.4 K_{28}$$

$$0 - 0.5 K_{28}$$

Each loading was carried out ten times. Two days later, the speed of increasing the force was raised:

$$0 - 0.2 K_{28}, \quad 0 - 0.4 K_{28}, \quad 0 - 0.6 K_{28}$$

A typical result is presented in Tables 1, which list the test results for specimens K-500-I. Each test made it clear that diminution in the Young's modulus by reason of repeated or increased loads is immaterial. The strength of concrete was found to increase with its age. This was not the case with Young's modulus, at least not in the investigations made in this way. Consequently, no motivation exists for the acceptance, without amendment, of behaviour models in which Young's modulus is made dependent upon the strength of

Table 1. Specimen K-500-I. Testing age 152 days.

$\sigma$	Number of loading	$E_A$	$E_B$	$E_C$	$\nu_A$	$\nu_B$	$\nu_C$
0.2 K <sub>28</sub>	1	2.78	2.68	2.86	0.184	0.157	0.199
	2	2.78	2.71	2.84	0.189	0.161	0.195
	3	2.80	2.71	2.82	0.193	0.163	0.196
	4	2.81	2.70	2.83	0.194	0.159	0.194
	5	2.79	2.71	2.84	0.190	0.167	0.199
	6	2.80	2.67	2.85	0.185	0.155	0.192
	7	2.79	2.70	2.79	0.181	0.162	0.194
	8	2.84	2.69	2.81	0.193	0.161	0.191
	9	2.78	2.67	2.79	0.183	0.155	0.194
	10	2.73	2.67	2.80	0.183	0.158	0.192
0.4 K <sub>28</sub>	1	2.88	2.77	2.91	0.194	0.175	0.204
	2	2.85	2.80	2.92	0.192	0.176	0.204
	3	2.84	2.77	2.91	0.196	0.176	0.205
	4	2.86	2.76	2.90	0.194	0.174	0.204
	5	2.87	2.76	2.89	0.195	0.177	0.203
	6	2.87	2.77	2.91	0.199	0.178	0.206
	7	2.88	2.78	2.91	0.196	0.178	0.205
	8	2.87	2.78	2.85	0.196	0.179	0.199
	9	2.85	2.76	2.86	0.199	0.178	0.203
	10	2.85	2.74	2.89	0.198	0.176	0.204
0.6 K <sub>28</sub>	1	2.89	2.81	2.91	0.204	0.188	0.211
	2	2.86	2.76	2.89	0.201	0.187	0.211
	3	2.78	2.78	2.88	0.197	0.186	0.214
	4	2.83	2.81	2.86	0.205	0.190	0.212
	5	2.83	2.77	2.85	0.201	0.188	0.213
	6	2.85	2.74	2.87	0.205	0.187	0.214
	7	2.84	2.76	2.86	0.204	0.188	0.214
	8	2.82	2.73	2.84	0.205	0.186	0.216
	9	2.82	2.74	2.84	0.207	0.188	0.215
	10	2.81	2.74	2.84	0.207	0.189	0.216
		2.83	2.76	2.87	0.204	0.188	0.214

concrete. It could rather be confirmed that for a fixed compression stress, Young's modulus retains its value as constant, independent of the way of conducting the measurements.

Consequently, it is quite another form of investigation to attempt to discover whether two concrete bodies, equal in strength, and of the same age, also always represent conforming values for Young's modulus, and in particular what this value amounts to in the case of concrete being loaded for the first time. With good reason, it may be expected that solution to this latter problem will be out of theoretical interest alone.

In strength classes K 275 and K 500, the concrete test specimens were 28 and 40 days old at the moment of measurement of Poisson's ratio. In strength class K 500 the investigation was repeated at the age of five months.

It becomes apparent from Figs. 3-8 and 9-15 that Poisson's ratio retained almost a constant value during the course of the investigation. In strength class K 500, the test specimens remained unloaded in the interval between 40 days and 5 months.

It thus follows that the question of the possible inconstancy of Poisson's ratio, in other words, whether this ratio can be influenced by continually repeated loadings, still awaits an answer. However, the above figures indicate that the influence of the resting time is of minor significance.

Young's modulus was measured in the same region as that in which was determined the strain in a direction orthogonal to the acting force. In the concrete bodies of the K 500 strength class, Poisson's ratio seems to be decreasing, while Young's modulus increases. The corresponding regression curves have been plotted in Fig. 16. The linear dependence valid between Poisson's ratio and

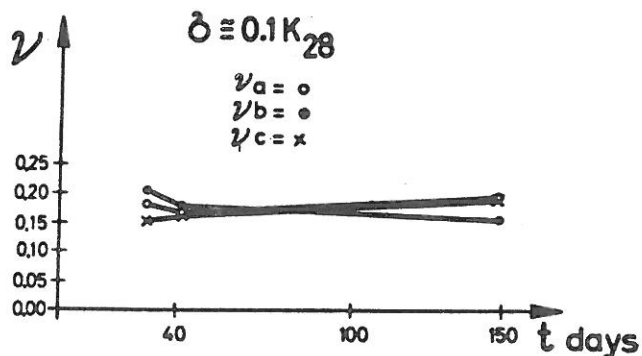


Fig. 3

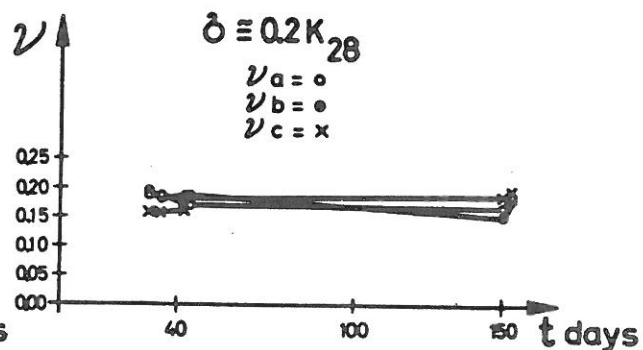


Fig. 4

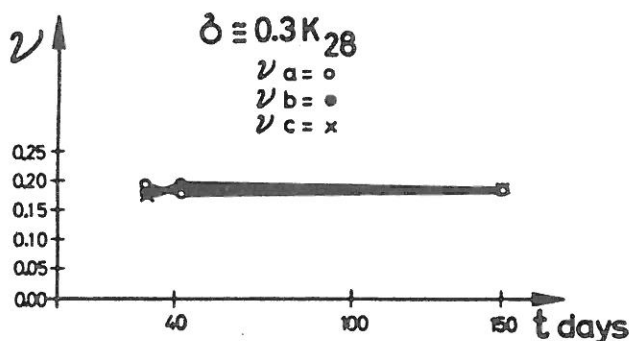


Fig. 5

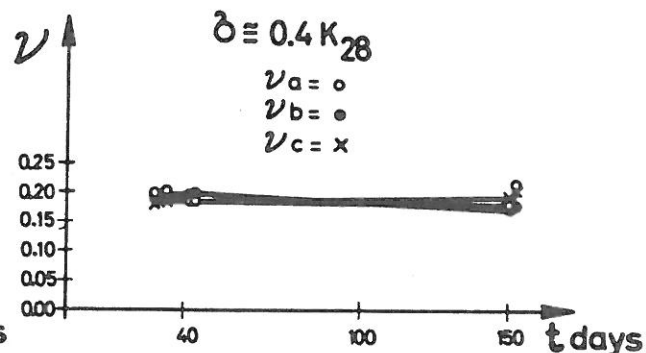


Fig. 6

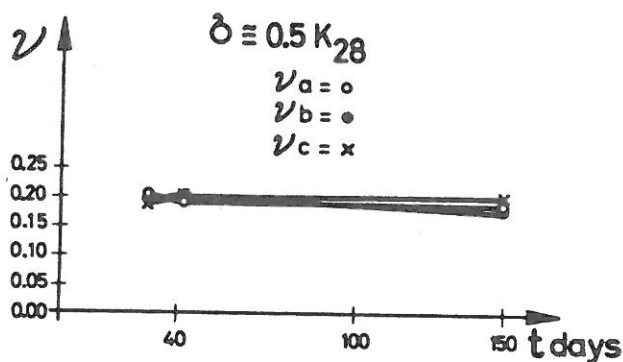


Fig. 7

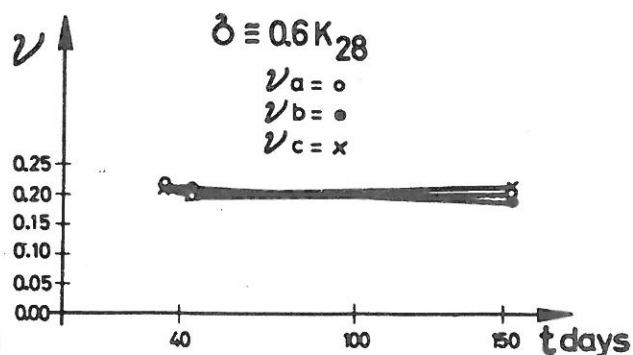


Fig. 8

Figs. 3-8. Graphs representing Poisson's ratio as a function of the testing time with the compressive stress changing within the limits  $0.1 K - 0.6 K_{28}$ . Strength class K 500. In figures 3-15 are these notations:  $\nu_a$  = upper part,  $\nu_b$  = central part,  $\nu_c$  = bottom part

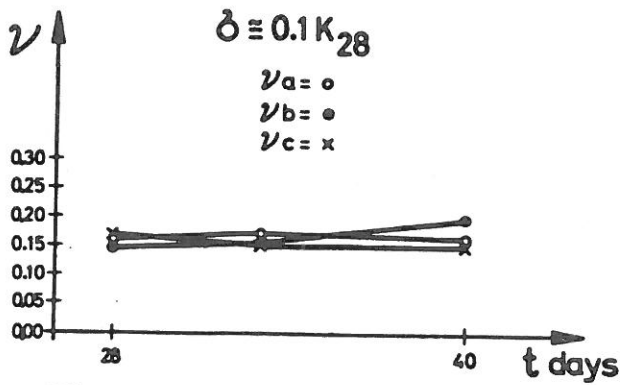


Fig. 9

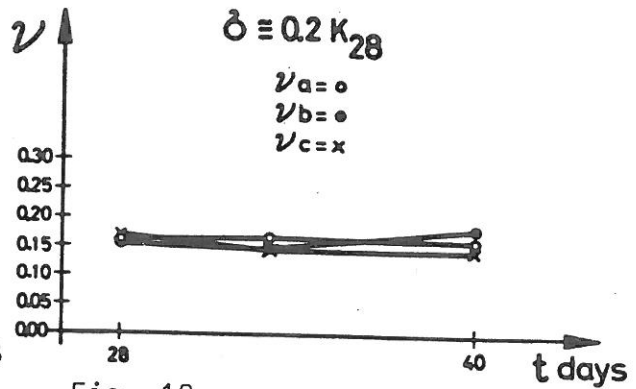


Fig. 10

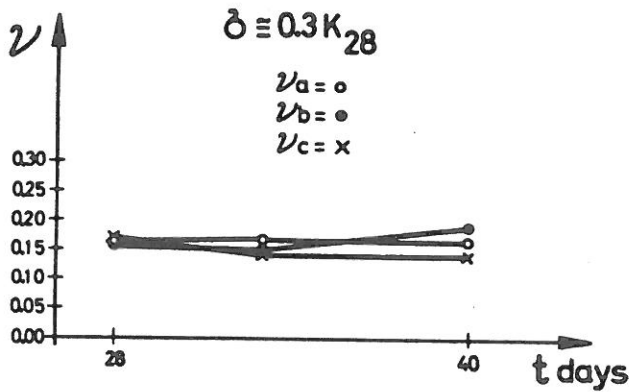


Fig. 11

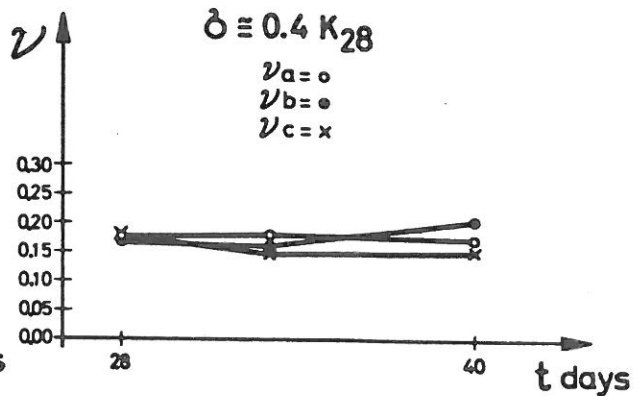


Fig. 12

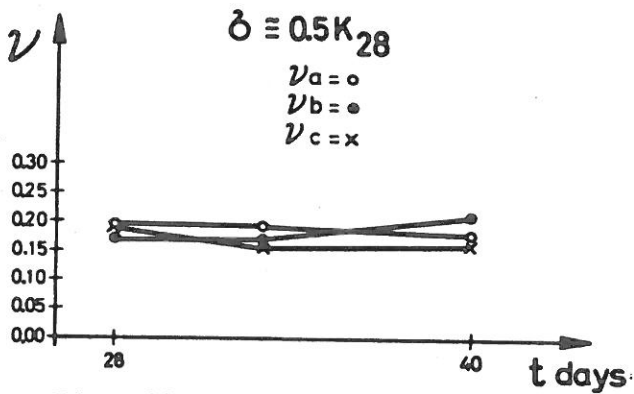


Fig. 13

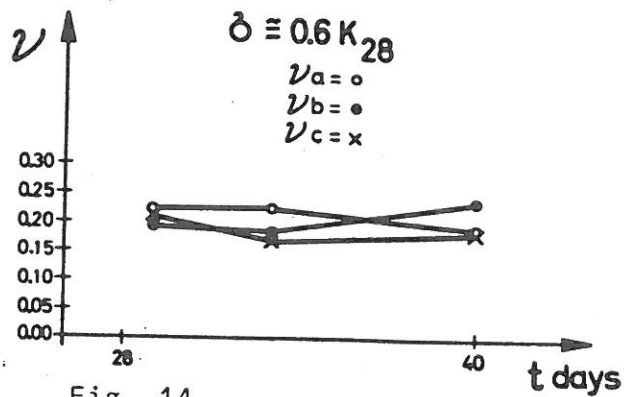


Fig. 14

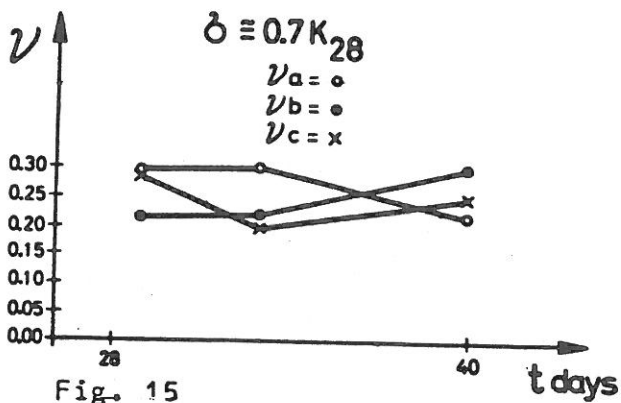


Fig. 15

Figs. 9-15.

Graphs representing Poisson's ratio as a function of testing time with the compressive stress changing within the limits  $0.1K_{28}$ - $0.7 K_{28}$ . Strength class K 275.

Young's modulus can be given by the formula

$$\nu_r = 0.22 - 0.125 E \cdot 10^{-6} \quad (\text{Fig. 16})$$

In strength class K 275, in which the same maximum aggregate grain size was used, this linear dependence was expressed in the form

$$\nu_r = 0.4 - 0.9 E \cdot 10^{-6} \quad (\text{Fig. 17})$$

On comparison with the now-rising Young's modulus, it becomes apparent that the diminution in Poisson's ratio was relatively more marked than in the the K 500 class.

## 6. POISSON'S RATIO AND THE STRENGTH OF CONCRETE

With the test specimens selected for this investigation series, the small number of values of cubic strength measured did not allow of any exact evaluation of the conditions of dependence valid between Poisson's ratio and the strength of concrete. Nevertheless, the regression curves indicate that the average value of this ratio was 0.18, and that the lowest value of 0.15 was passed no more than slightly.

## 7. POISSON'S RATIO AND STRESS

It becomes apparent from the tables given below that, if the load is increased, and if the compression stress is changed within the interval 0.1 - 0.6, the scale in which variations occurred in the values of Young's modulus was rather concise. To the contrary, Poisson's ratio increased alongside growing stresses. In respect of the concrete material in strength class K 500, it could be confirmed that here the dispersion was less among the values of Poisson's

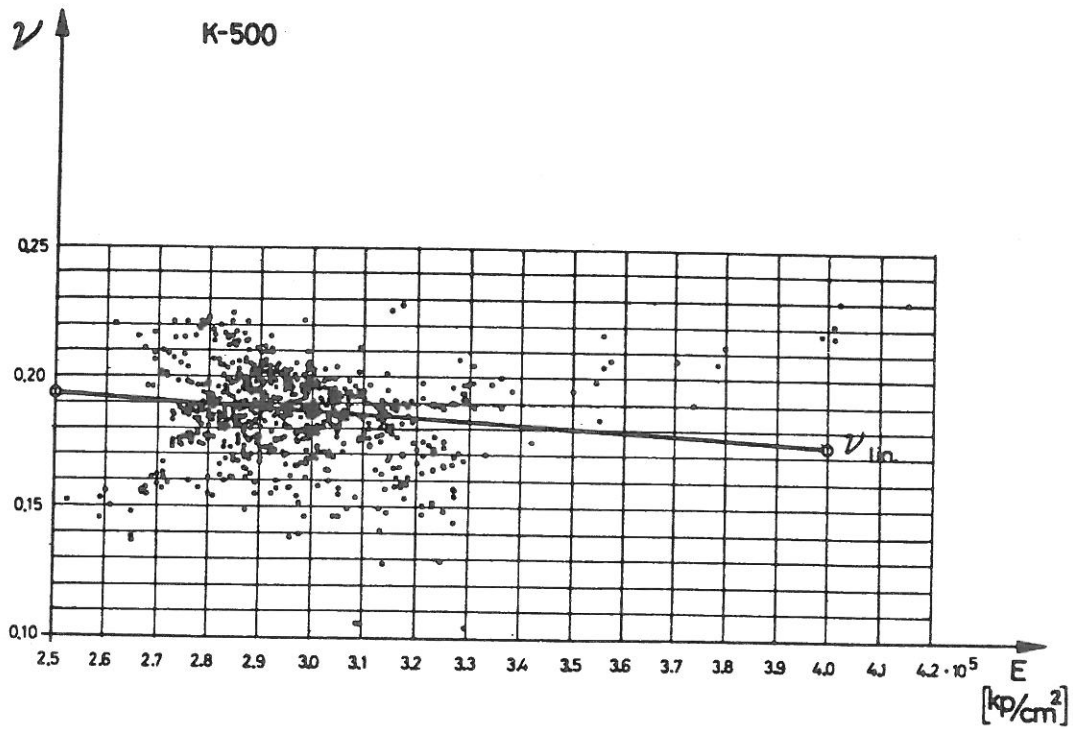


Fig. 16. Dependence between Poisson's ratio and Young's modulus, for concrete of strength class K 500.

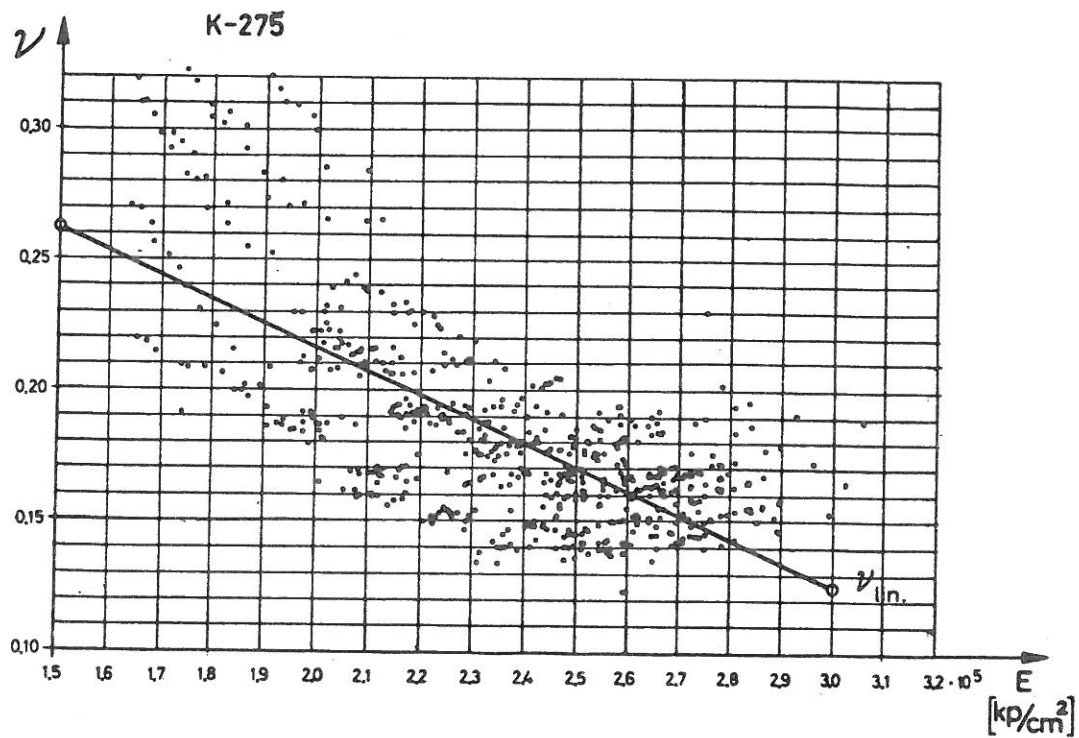


Fig. 17. Dependence between Poisson's ratio and Young's modulus, for concrete of strength class K 275.

ratio measured than was the case in class K 275. This may be attributable to the amount of cement lime in concrete.

8. POISSON'S RATIO UNDER THE INFLUENCE OF VARIOUS LOADINGS

If, with a view to discovering the Young's modulus proper, and what the strain is, a concrete body is taken under consideration, attention is also due to how the body will be loaded, and also to whether the specimen will be loaded for the first time. The necessity of this became obvious from the reported series of loadings alone.

The strain in the direction of loading provides a good example of this. Independent of the state of stress in concrete, at the second time of loading the strain indicated almost a constant value. This also applied in cases in which the loading slightly exceeded its previous maximum value. It is apparent that on the first occasion the loading attained a value of  $0.5 K_{28}$ , and on the second,  $0.6 K_{28}$ . However, the value of Poisson's ratio changed perfectly in the same manner as at the first loading time: the Poisson's ratio rose in accordance with increasing stress.

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