ANALYSIS AND DESIGN OF REINFORCED CONCRETE STRUCTURES IN THE SOVIET UNION

HENDRIK LAUL

Rakenteiden Mekaniikka Vol. 2 No. 3 1969 ss. 96—102; Kustannusosakeyhtiö Rakennustekniikka, Helsinki

The length of this article makes it impracticable to give in detail all the regulations applied nowadays in the Soviet Union for the design of reinforced concrete structures. Accordingly, an attempt is made below to elucidate the most important principles which, I think, differ from those prevalent in other countries.

In the analysis of reinforced concrete structures (as in that of structures made of other building materials), three points need to be checked:

1) the strength of the structure as a whole, and in detail;
2) the stiffness of the structure (deflections);
3) possible local disturbances (such as the width of cracks).

1. Checking the strength

Discussion is concerned first with the factor of safety. In the Soviet Union the so-called »system of differentiated factors of safety» (introduced in 1959) is applied. Allowable stresses have not been defined at all. Instead of this an endeavour is made to compare the minimum strength (determined statistically) of some section loaded by some maximum internal force T (bending moment M, normal force N, shearing force Q etc.), induced by some statistically possible, and most disadvantageous combination of external loadings. In general, this idea is expressed by the following inequality:

$$\Sigma n_i \overline{T}_i \leqq \Phi(S; k_a\overline{R}_a; k_b\overline{R}_b; m_1; m_2; \ldots) \qquad (1)$$

Here, on the left-hand side is the statistically possible maximum internal force $T(M,N,Q)$, the so-called design force. $\overline{T}_i$ are internal forces attributable to various working loads (such as working dead load, live load, temperature, and so on) which are taken from stipulations, or which we calculate or check ourselves.

To arrive at the statistically possible maximum internal force T, the working forces $\overline{T}_i$ are multiplied by the corresponding overloading factors n_i. In general, the overloading factor has been established for the dead load as $n_g = 1.1$, and in most cases for a live load $n_p = 1.4$, since live loads are exposed to much greater deviations than is the dead load.

The right-hand side of formula (1) determines the least possible strength of the sections, which is a function of the quantities given in brackets.

S is a geometrical quantity of the section determining the strength (e.g. if the internal force is a normal force N, S is a cross-sectional area F; if the internal force is a bending moment M, S is some moment of resistance and so on).

$\overline{R}_a$ and $\overline{R}_b$ respectively are the standard strength of concrete (various kinds of compression strengths, tensile strengths, etc.) which can be arrived at from stipulations, or found experimentally by ourselves. As the so-called factors of homogeneity k_a and $k_b \leqq 1$, the real strengths of material in a structure $R_a = k_a\overline{R}_a$ and $R_b = k_b\overline{R}_b$ (called design strengths) can be statistically smaller than the standard strengths $\overline{R}_a$; $\overline{R}_b$. The factors k_a, k_b have been determined by extensive statistical research work during the course of many years. It is assumed that the probability of a still smaller strength is about 1: 1000, then generally we get for reinforcement $k_a = 0.9$; but for concrete only, the figure is about $k_b = 0.5$, since deviation in the strength of concrete far exceeds that of steel.

Two extreme cases are now treated. First, let us have a case in which the resistance of the section is defined by the resistance of concrete only (e.g. a weakly-reinforced column), and the predominant load is live load p, dead load $g \approx 0$. Then formula (1) gives (somewhat simplified)

$$\overline{N}_p \cdot n_p \leq F_b \cdot k_b \cdot \overline{R}_b, \text{ or } \frac{\overline{N}_p}{F_b} \leq \frac{\overline{R}_b}{k}$$

where the factor $k = n_p : k_b = 1.4 : 0.5 = 2.8$ could be considered as the usual factor of safety.

However, if this is the other way round, the resistance of the section is defined by the strength of steel, and the predominant load is dead load g, live load $p \approx 0$ (for instance, some roof girders with very large spans):

$$\overline{M}_g \cdot n_g \leq F_a \cdot z(k_a \overline{R}_a)$$

or

$$\frac{\overline{M}_g}{F_n \cdot z} \leq \frac{\overline{R}_a}{k}$$

where $k = n_g : k_a = 1.1 : 0.9 = 1.22$;
z = internal lever arm
F_a = area of tensile steel

As can be seen, the usual factor of safety k in these extreme cases differs more than twice (2.8 and 1.22).

Of course, in most cases the resistance of the section depends on both the materials concerned, and both dead and live loads may vary within smaller limits. In general, if the dead load dominates, and the resistance of the section is defined by the strength of steel, the design in accordance with the system of factors of safety applied in the Soviet Union, may provide a greater saving than do the results arrived at by application of the usual factor of safety.

The overloading factors n_i, and the factors of homogeneity k_i, are the basic factors. The other factors m_i in formula (1) are equal to one in most cases. However, on occasion an attempt is made to take into account special conditions of materials and structures, and to make computing more adequate to the real working measure of the structure.

Of course, the geometrical quantity S is also subjected to some statistical deviations. For example, the diameters of reinforcement bars may differ slightly from those marked in the drawings. However, this influence is not considered in the Soviet stipulations.

When compression members are being analysed, it is necessary to consider the influence of buckling and plastic flow (induced by dead load). Only lately has this effect been introduced as a factor which increases the dead load.

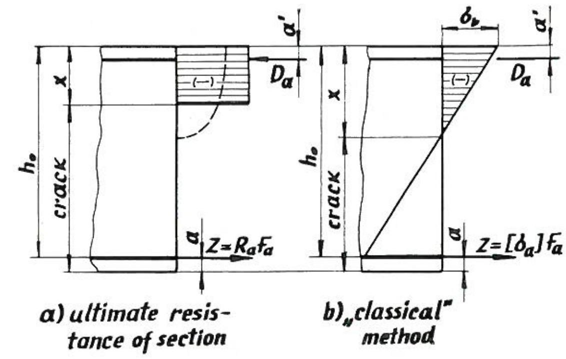

Fig. 1. Stress diagrams

The section of a reinforced concrete beam is designed on the assumption that the ultimate resistance of the section is achieved (introduced in 1938). As a rule, the failure starts (fig. 1) when the stresses in the tension steel achieve the yield point, with the shape of the compressive stress diagram being almost a parabola (shown in fig. 1 a by a dotted line). The further increase in loading results in the appearance of cracks, and just before the failure, the compressive stress diagram is very nearly rectangular. The distance x (the height of the compression zone) acquires the minimum value, which means that the internal lever arm of the section z will be maximum. Thus the normal failure ends in the failure of the compression zone of the beam. The steel in the compression zone also achieves its yield point (if it does not exceed 4200 kg/cm²). If this situation is compared with the so-called »classical method» where materials are presumed to remain elastic, and plain sections to remain plain until the end, the following differences are observable:

1) Concrete and steel achieve their ultimate strengths simultaneously, whereas when the »classical method» is applied this situation would be rather rare and even uneconomical.

2) It is possible that the need of the tension steel is smaller, since the lever arm z is longer. However, as a rule the increase in the lever arm is not considerable, and therefore the area of tension steel does not differ very much from that obtained by use

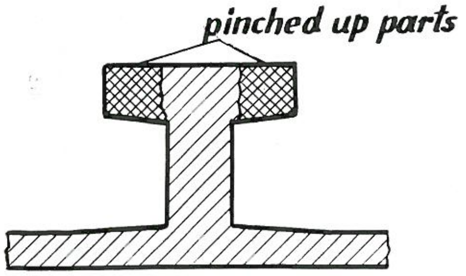

Fig. 2. The section of a „Finnish" beam

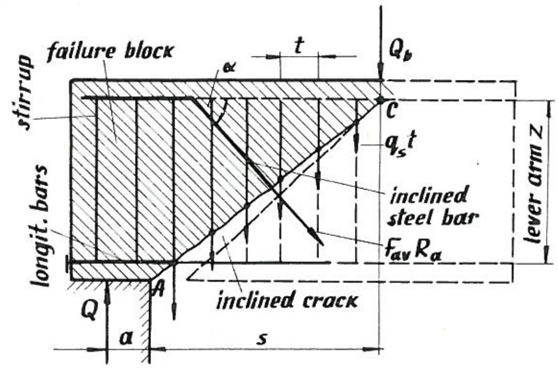

Fig. 3. The end region of a beam

of the »classical method» (the differences in the factor of safety system are ignored here).

3) The design according to the »classical method» often calls for double reinforcement, but if the ultimate strength method is applied, the compression reinforcement can very often be omitted. For instance, in Estonia it was possible to use »Finnish beams» (fig. 2 — the compression parts are strengthened by flanges) with flanges distorted by fire during the Second World War without strengthening them.

4) The number of equations is less than that in the »classical method», since the compatibility conditions of strains resulting from the assumption of plain sections cannot be used. The designer has consequently freer hands.

When the tension part of a beam is overreinforced, it may happen that the failure starts in the compression zone. The failure is a brittle one, and not plastic, as in the case of a normally reinforced beam. As numerous experiments have demonstrated, a failure is normal, when $x \leqq 0.55\ h_o$, with h_o being the depth to tension steel.

Axially-loaded and normally-reinforced compression members (such as columns) designed in accordance with the ultimate strength of the section do not as a rule differ much from those designed by use of the »classical method» (notwithstanding the assumtion that the stresses in compression steel reach the yield point). However, an eccentrically-loaded member could on occasion need much less reinforcement in its compression zone, if it is designed by the ultimate resistance method. As in the case of a double reinforced beam, the rectangular shape of the compressive stress diagram is the main reason for this pheonomenon. Another important difference is that buckling is taken into account not by application of the usual buckling factor which increases the real compressive force N, but by means of a factor which increases the eccentricity of the compressive force $e_o = M : N$. Of course, this fits in better with the elastic theory of buckling.

In the phase of ultimate strength, a »failure block» will form (hatched in fig. 3) at the end of the beam. This tends to separate from the rest of the beam along the inclined crack A—C. From the equilibrium of the vertical forces, applied to the failure block, it follows that (fig. 3)

$$Q \leqq F_{av} R_a \sin\alpha + g_s s + Q_b \qquad (2)$$

where Q — maximum shearing force, induced by design loads;

g_s — ultimate tensile strength of stirrups, calculated on a unit length of the beam;

Q_b — »shear» resistance of the compression part of the section;

s — horizontal length of the inclined crack.

Here

$$g_s = \frac{nR_a f_s}{t} \quad (3)$$

where n — number of stirrups in a section;
f_s — area of one stirrup;
t — distance between the stirrups along the beam;
R_a — ultimate design tensile strength of stirrup steel.

The shearing resistance of the compression part has been found experimentally to be

$$Q_b = \frac{0.15\ R_b\, bh^2_o}{s} \quad (4)$$

The influence of longitudinal tension reinforcement has been omitted from formula (2), since the »dowel-effect» is very small in the ultimate phase.

It becomes evident from formula (2) that the reinforcement in stirrups has greater utility than that in inclined bars (this contradicts the results of the »classical» theory).

Consequently (and to some extent also with a view to the advanges in the erection), inclined bars have recently been used very seldom in the Soviet Union. Thus in most cases, the shearing force Q is resisted only by the stirrups and Q_b.

In this case, formula (2) gives

$$Q \leq g_s\, s + \frac{0.15\ R_b\, bh^2_o}{s} \quad (2')$$

Evidently, here is the minimum resistance of the shear force dependent upon the »s». If this simple problem is solved, there is derived the extreme case of

$$\min\ Q \leq \sqrt{0.6\ R_b\, bh^2_o\, g_s} \quad (5)$$

From formula (5), the necessary force of the stirrups is immediately determinable.

Of course, another question concerned with a failureblock may arise. In a ultimate phase, it is possible that the bending moment in point C is not resisted, by reason of improper anchorage of the longitudinal reinforcement into the failure block. In the Soviet Union, the minimum anchorage length of reinforcement at a support is laid down. If this is guaranteed, the checking of this bending moment in an inclined crack can be omitted. However, if no anchorage length is stipulated, the whole necessary force in longitudinal bars is to be met by special anchorage. Consequently, in the design of usual reinforced concrete structures, the bond stresses are not examined at all, and even the bond strength is not determined (except in the case of aerated concrete structures). It may be that the inclined crack does not begin just at the support, but at a distance »a» from it, where the bending moment $Q \cdot a$ would induce cracking in the same section. Evidently the disregard of this fact is a mistake advantageous to safety.

It is interesting to note that Q_b does not depend upon the existence of flanges in the compression part of the section (T-sections), nor upon the existence of some normal force (such as prestressing).

On a general examination, the resistance to the shearing force Q is much simpler than in »calssical methods».

In addition, it provides more economical solutions in most cases. In general, a marked discrepancy is concealed everywhere, including here. As a rule, the internal forces (M, N, Q etc.) are computed with the structure being regarded as an elastic body (this is especially important in statically indeterminate structures). However, the design of the sections is made on the assumption of the stage of ultimate resistance. It is clear that the distribution of internal forces can be quite different from that in an elastic body. A great deal of research work has been undertaken to remove this discrepancy. In the Soviet Union it is allowable that continuous beams and slabs be analysed on the assumption that enough plastic hinges have appeared (or plastic line hinges in the case of a plate or a shell) in the structure to make it statically determinate. Nevertheless, »ultimate load methods» are not permissible in the design of main girders, and some special structures (such as reservoirs). In many cases slabs, continuous in two directions, are also analysed by the ultimate load method. For instance, the ultimate load q_p for a rectangular slab, continuous in two directions (fig. 4), is

$$q_p \leq \frac{24}{l_2^2 \, (3\, l_I - l_2)} \, (\overline{M}_I + \overline{M}_2 + \overline{M}_I + \overline{M}_{II}) \tag{6}$$

where $\overline{M}_I$, $\overline{M}_{II}$, $\overline{M}_I$, $\overline{M}_2$ are ultimate bending moments on the total length of the corresponding line hinges (e.g. $\overline{M}_I = F_{aI} \cdot R_a z$ where F_{aI} is the whole area of the reinforcement crossing that line hinge).

Formula (6) is based on the assumptions:

1) Line hinges have a form as shown in fig. 4. On the supports, the line hinges will open at the top (shown by full lines), and in the field at the bottom (shown by dotted lines).

2) The stresses in the crossing reinforcement reach yield point simultaneously.

3) The elastic deformations of the slab are disregarded.

If the line hinge C—D obtains the virtual deflection $w = 1$, then the angles between two adjacent sections in every line hinge are determined. Now, by application of the principle of virtual displacements, i.e. by making the work of external load q_p equal the work of ultimate bending moments in all line hinges with the corresponding virtual displacements $w = 1$, formula (6) is derived. Formula (6) has an important advantage. The reinforcement of the slab is not so strictly stipulated as it is in application of the theory of elasticity. Consequently, no benefit is gained by reducing the reinforcement in the outer quarters of the slab, as is usually done. Quite analogically, it is possible also to analyse flat slab floors, and even some kinds of shells. Resently attempts have been made to analyse cylindrical shells by means of linear programming.

Prestressed reinforced concrete structures are also analysed analogically. Naturally, in the state of ultimate loads all the effectiveness of the prestressing is lost by reason of the plastic deformations, and (almost) all the differences disappear as compared with usual reinforcement concrete. It is known that one of the most important advantages of prestressed concrete is that the prestressing increases the resistance of the structure to shearing force Q. However, this advantage disappears completely in the stage of ultimate loads. It has been established experimentally that Q_b (see fig. 4) does not change as a consequence of prestressing, and this also applies if some prestressing force can be kept in the reinforcement of the compression zone at the state of the ultimate load. In normal reinforced concrete members, cracks are allowed to appear, but in most cases the prestressing force must be capable of preventing cracks under working loads. Of course, the effective prestressing force depends largely upon its general loss. In the Soviet Union the general loss is calculated by the aid of single losses induced by shrinkage, plastic flow, relaxation of prestressing steel, friction between the steel and the tubes, relaxation of the prestressed steel following the increase in temperature during the hardening process, the influence of later prestressed steel, yielding of the anchors of the bars, and so on. The general

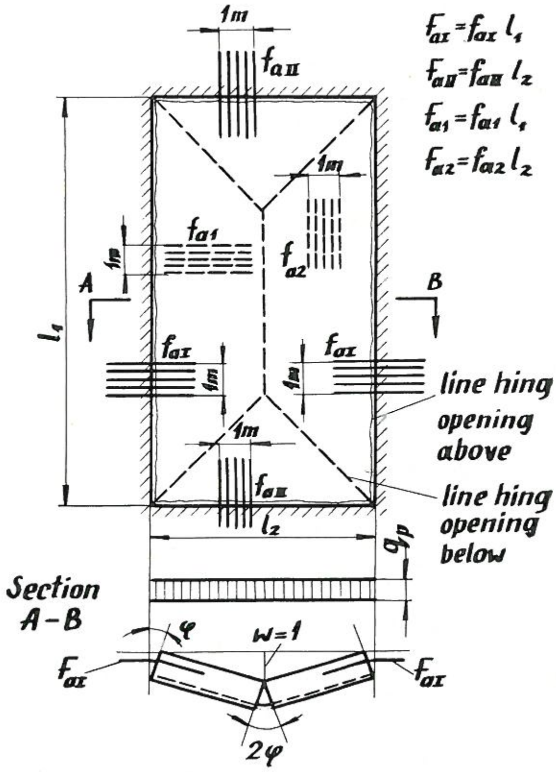

Fig. 4. A slab with the failure cracks

loss is arrived at by summation of all the single losses. On occasion, it may even exceed 4000 kg/cm², if the initial prestress has been 10 000 kg/cm². It is practicable to check the general loss of prestressing at various moments of time. However, it is impracticable to calculate this very precisely. An attempt has been made to determine the probable maximum and minimum values of the general loss. The minimum is used if we wish to examine the resistance of the member to the prestressing force alone, and the maximum, if we wish to discover whether the remaining prestress is capable of preventing cracks under working conditions. Prestressed concrete beams are also, as a rule, examined in measures for the avoidance of inclined cracks arising near the supports. This is effected in greater analogy with the »classical methods».

2. Flexural stiffness of beams

Earlier, the flexural stiffness of reinforced concrete beams did not pose a serious problem at all, since the necessary stiffness was always guaranteed. Of course, it was sometimes necessary to calculate the stiffness $B = EI$, when the task was that of designing some special, statically indeterminate structure, or a structure under a dynamic load. Nowadays, however, with the widespread use of prefabricated reinforced and prestressed structures, examination of the stiffness of a beam has grown into an important part of the design. In the Soviet Union, the deflections of the beams are always calculated with working loads ($\bar{q}$). They can be then compared with the allowable deflections.

Instead of the well-known flexural stiffness of a beam given by the strength of materials such as EI (although this is used for prestressed concrete beams with small changes), the following formula (given in simplified form) is applied in the Soviet Union for a beam with a rectangular section, and reinforced only in the tension zone:

$$B = (zE_aF_a) : (\varphi_a : h_o) + (0.9\ n\mu : x) \qquad (7)$$

which takes into account the existence of cracks in the tension zone of a beam.

In formula (7), the following symbols have been used: h_o, z, and x respectively are the depth to the reinforcement, the internal lever arm, and the depth of the compression zone; φ_a is a factor that takes into account the contribution of concrete between cracks; ν is a factor concerned with the influence of plastic flow (when the loads are of short duration, it is 0.5; when the loads acting over a lengthy period, ν varies from 0.10 to 0.20, dependent upon the humidity of the surroundings); $n = E_a : E_b$; $\mu = F_a : bh_o$. There are obtained

$$x \approx (10\ \mu n h_o) : [1 + 18 + \mu n + (5\bar{M} : bh_o^2\, \bar{R}_b)] \qquad (8)$$

$$z = h_o\, [1 - 0.5\, (x : h_o)]$$

$$\varphi_a = [1.3 - (0.23\ bh^2\, \bar{R}_p) : \bar{M}] \leqq 1$$

where $\bar{R}_p$ and $\bar{R}_b$ respectively are the working (standard) tensile and compressive strengths of concrete, and $\bar{M}$ the bending moment from the working load.

By the application of formula (7), it is possible to arrive at stiffnesses which are only one half or even less of the usual EI.

Of course, analogical formulae exist for calculation of the stiffness of beams with sections of different shape, or with reinforcement also in the compression zone. It is also possible to consider the existence of a longitudinal force.

3. Local disturbances

The width of cracks is of importance if an attempt is made to estimate the durability of steel in concrete. In Soviet Union, cracks of 0.3 mm width are in most cases considered as still permissible under working loads. A theory for forecasting the width of cracks has been elaborated. To simplify explanation, the following example of a tension member under a working normal force $\bar{N}$ is given. The beams are treated quite analogically.

The width of a crack can be determined from formula

$$a_T = \varepsilon_a\, l_T = (\varphi_a\, \sigma_a\, l_T) : E_a \qquad (9)$$

if the elongation of the concrete is ignored. Then the whole elongation of the steel between two cracks is taken for the width of

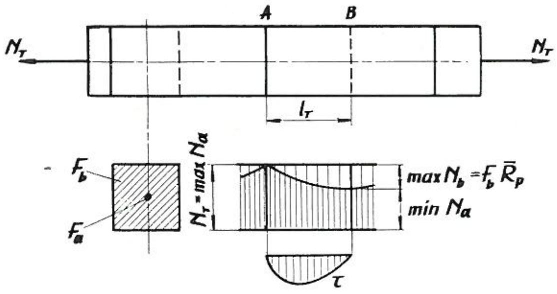

fig. 5. A tensile member under longitudional forse N_T, just causing cracks

the crack. In formula (9), φ_a is the same factor as in formula (8). In this special case it is

$$\varphi_a = [1 - (0.7\, N_T) : \bar{N}] \leqq 1 \qquad (10)$$

if the loads are of short duration, and

$$\varphi_a = [1 - (0.35\, N_T) : \bar{N}] \leqq 1 \qquad (10')$$

if the loads are acting for a long time.

In formula (10), N_T is the normal force that induces cracks:

$$N_T \approx 1.8\, F_b \bar{R}_p$$

In formula (9), the stress of reinforcement in the cracked section A, under the working load, is

$$\sigma_a = \bar{N} : F_a$$

The distance between two cracks l_T is found as follows. Fig. 5 illustrates the situation in which crack A has just arisen under the external force N_T, and the adjacent crack B is just appearing. A part of the whole tensile force in crack A (max. N_a) is then given over to the concrete by bond stresses between concrete and steel along the distance between two cracks. In the section B, the maximum tensile force in concrete is max $N_b = F_b \bar{R}_p$.

The equation for finding l_T is obviously

$$F_b \bar{R}_b = \omega \cdot \tau \cdot s \cdot l_T$$

The left-hand side represents the ultimate tensile strength of the concrete section (the new cracks are just arising), and the right hand side is the ultimate bond force. Factor ω represents an attempt to consider the shape of the diagram of bond stress (for rectangular $\omega = 1$; for triangular $\omega = 0.5$). s is the total perimeter of all the reinforcing bars.

If we put $u = F_a{:}s$, and assume $\bar{R}_b{:}\ \omega\tau \approx 1$, then $l_T = u{:}\ \mu$ is derived. If deformed bars are used, l_T is 30 % less.

It is discernible that the larger the concrete section is at a given reinforcement, the less is μ, the larger is l_T and the larger is a_T. Thus the use of large concrete sections is not desirable even from this aspect (of course, except in regard to cases in which cracks do not arise at all).

4. Conclusions

This short article contains no more than the main points of the analysis of reinforced concrete structures in the Soviet Union, and even then in very concise form. Many questions dealt with have not been touched upon at all, although in the Soviet Union very precise stipulations exist for the analysis of structures

— working steadily at high temperature (chimneys)
— working under loads of high frequency (machine foundations)
— working in the seismic regions
— loaded in a combination of bending and twisting moments, and so on.

In this article many stipulations have not been remarked upon such as the design of a reinforced concrete structure after it has been calculated. Naturally, the widely used prefabrication of reinforced concrete structures has introduced many changes in the design of details, aimed at making a structure more suitable for industrial production methods, which have been brought into use on a major scale since 1954. Since then, many achivements have been recorded, and, as is usual with innovations some ill fortune as well.

Research work concerned with improvement in the analysis of reinforced concrete structures is still in progress, and a number of facts put forward in this article may not be valid some years later.

A large number of scientists have worked for many years on the questions described above. These include A. A. Gvozdyev, V. V. Michailov, V. I. Murashev, and M. S. Borishansky.

Hendrik, Laul, Professor in structural engineering, Techical University in Tallinn