Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävä 7:

Pinnan parametrimuotoinen yhtälö on

$$\left.\begin{aligned} x &= a(1-\xi+\xi\cos\phi) \\ y &= a\xi\sin\phi \\ z &= 2a(1-\xi) \end{aligned}\right\},\ 0\le\varphi\le 2\pi,\ 0\le\xi\le 1$$

Olkoon $\alpha=\xi$ ja $\beta=\phi$. (a) Konstruoi pinta (oikeakätiseen) $x,y,z-$koordinaatistoon, jonka $z-$akseli on ylöspäin, piirtämällä seuraavat $\xi-$viivat: $\phi=0$, $\phi=\pi/4$, $\phi=\pi/2$, $\phi=3\pi/4$, $\phi=\pi$, $\phi=5\pi/4$, $\phi=3\pi/2$ ja $\phi=7\pi/4$ sekä seuraavat $\phi-$viivat: $\xi=0$, $\xi=1/4$, $\xi=1/2$, $\xi=3/4$ ja $\xi=1$. Voit myös piirtää pinnan matematiikkaohjelmalla. (b) Määritä mittakaavatekijät A ja B, koordinaattiviivojen välinen kulma χ, yksikkönormaalivektori **n,** kaarevuussäteet R_ξ, R_ϕ ja kierevyyssäde $R_{\xi\phi}$, pääkaarevuussäteet R_1 ja R_2 sekä pääkaarevuuksien suunnat määrittelevät parametrit λ_1 ja λ_2. (c) Selvitä kuoren tyyppi ja tulkitse kaarevuuksille sekä pääkaarevuuksille ja niiden suunnille saamiasi tuloksia erityisesti pisteissä P: $\xi=1/2,\ \phi=0$, Q: $\xi=1/2,\ \phi=\pi/2$ ja R: $\xi=1/2,\ \phi=\pi$.

Osittainen vastaus:
(b)

$$R_\xi=\infty,\ R_\phi=\frac{\sqrt{6-2\cos\phi-\sin^2\phi}}{2}a\xi,\ R_{\xi\phi}=\infty,$$

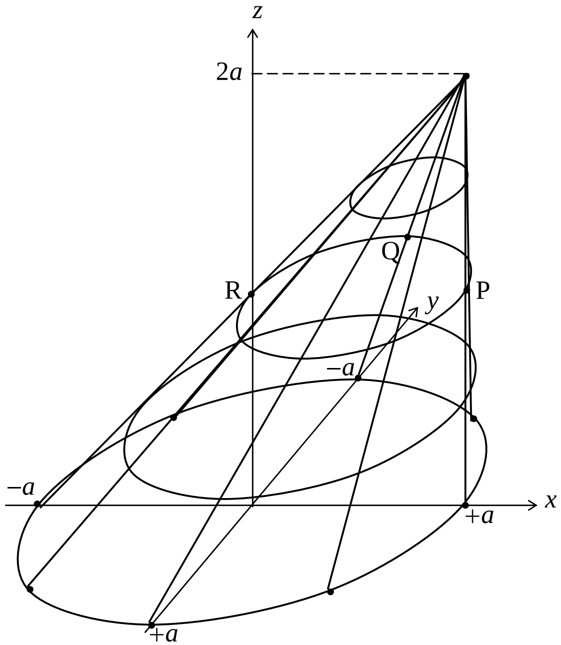
$$R_1=\frac{(6-2\cos\phi-\sin^2\phi)^{3/2}}{2(6-2\cos\phi)}a\xi,\ R_2=\infty,\ \lambda_1=-\frac{1}{\xi}\frac{6-2\cos\phi}{\sin\phi},\ \lambda_2=0.$$

Palautus pe 5.11 klo 16.00 mennessä.

Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävän 7 ratkaisu:

(a)



(b)

Paikkavektori ja sen derivaatat:

$$\mathbf{r} = a(1-\xi+\xi\cos\phi)\mathbf{i} + a\xi\sin\phi\mathbf{j} + 2a(1-\xi)\mathbf{k},$$
$$\mathbf{r},_{\xi} = a(-1+\cos\phi)\mathbf{i} + a\sin\phi\mathbf{j} - 2a\mathbf{k},$$
$$\mathbf{r},_{\phi} = -a\xi\sin\phi\mathbf{i} + a\xi\cos\phi\mathbf{j},$$
$$\mathbf{r},_{\xi\xi} = \mathbf{0},$$
$$\mathbf{r},_{\phi\phi} = -a\xi\cos\phi\mathbf{i} - a\xi\sin\phi\mathbf{j},$$
$$\mathbf{r},_{\xi\phi} = -a\sin\phi\mathbf{i} + a\cos\phi\mathbf{j}.$$

Mittakaavatekijät ja koordinaattiviivojen välinen kulma:

$$\underline{A} = |\mathbf{r},_{\xi}| = a\sqrt{(-1+\cos\phi)^2 + \sin^2\phi + 4} = \underline{a\sqrt{6-2\cos\phi}},$$

$$\underline{B} = |\mathbf{r},_{\phi}| = a\xi\sqrt{\sin^2\phi + \cos^2\phi} = \underline{a\xi},$$

$$\begin{aligned}
\underline{\cos\chi} &= \frac{1}{AB}\mathbf{r},_{\xi}\bullet\mathbf{r},_{\phi} \\
&= \frac{1}{AB}[a(-1+\cos\phi)\mathbf{i} + a\sin\phi\mathbf{j} - 2a\mathbf{k}]\bullet(-a\xi\sin\phi\mathbf{i} + a\xi\cos\phi\mathbf{j}) \\
&= \frac{1}{AB}[\sin\phi(1-\cos\phi) + \sin\phi\cos\phi] = \frac{a^2\xi}{AB}\sin\phi \\
&= \underline{\underline{\frac{\sin\phi}{\sqrt{6-2\cos\phi}}}}
\end{aligned}$$

$$\sin\chi = \sqrt{1-\cos^2\phi} = \sqrt{1-\frac{\sin^2\phi}{6-2\cos\phi}} = \sqrt{\frac{6-2\cos\phi-\sin^2\phi}{6-2\cos\phi}}$$

Yksikkönormaalivektori:

$$\begin{aligned}
\mathbf{n} &= \frac{1}{AB\sin\chi}\mathbf{r},_{\xi}\times\mathbf{r},_{\phi} \\
&= \frac{1}{AB\sin\chi}[a(-1+\cos\phi)\mathbf{i} + a\sin\phi\mathbf{j} - 2a\mathbf{k}]\times(-a\xi\sin\phi\mathbf{i} + a\xi\cos\phi\mathbf{j}) \\
&= \frac{a^2\xi}{AB\sin\chi}[(-1+\cos\phi)\mathbf{i} + \sin\phi\mathbf{j} - 2\mathbf{k}]\times(-\sin\phi\mathbf{i} + \cos\phi\mathbf{j}) \\
&= \frac{1}{\sqrt{6-2\cos\phi}\sin\chi}[(-1+\cos\phi)\cos\phi\overbrace{\mathbf{i}\times\mathbf{j}}^{\mathbf{k}} - \sin^2\phi\overbrace{\mathbf{j}\times\mathbf{i}}^{-\mathbf{k}} + 2\sin\phi\overbrace{\mathbf{k}\times\mathbf{i}}^{\mathbf{j}} - 2\cos\phi\overbrace{\mathbf{k}\times\mathbf{j}}^{-\mathbf{i}}] \\
&= \frac{1}{\sqrt{6-2\cos\phi-\sin^2\phi}}[2\cos\phi\mathbf{i} + 2\sin\phi\mathbf{j} + (1-\cos\phi)\mathbf{k}]
\end{aligned}$$

Kaarevuudet ja kierevyys:

$$\underline{\frac{1}{R_{\xi}}} = -\frac{1}{A^2}\mathbf{n}\bullet\overbrace{\mathbf{r},_{\xi\xi}}^{\mathbf{0}} = \underline{0},$$

$$\underline{\frac{1}{R_\phi}} = -\frac{1}{B^2}\mathbf{n}\bullet\mathbf{r},_{\phi\phi}$$

$$= -\frac{1}{B^2}\frac{1}{\sqrt{6-2\cos\phi-\sin^2\phi}}[2\cos\phi\mathbf{i}+2\sin\phi\mathbf{j}+(1-\cos\phi)\mathbf{k}]\bullet(-a\xi\cos\phi\mathbf{i}-a\xi\sin\phi\mathbf{j})$$

$$= \frac{1}{B^2}\frac{2a\xi(\overbrace{\cos^2\phi+\sin^2\phi}^{1})}{\sqrt{6-2\cos\phi-\sin^2\phi}} = \underline{\frac{2}{\sqrt{6-2\cos\phi-\sin^2\phi}}\frac{1}{a\xi}},$$

$$\underline{\frac{1}{R_{\phi\xi}}} = \frac{1}{AB}\mathbf{n}\bullet\mathbf{r},_{\phi\xi}$$

$$= \frac{1}{AB}\frac{1}{\sqrt{6-2\cos\phi-\sin^2\phi}}[2\cos\phi\mathbf{i}+2\sin\phi\mathbf{j}+(1-\cos\phi)\mathbf{k}]\bullet(-a\sin\phi\mathbf{i}+a\cos\phi\mathbf{j})$$

$$= \underline{0}$$

Pääkaarevuudet:

$$H = \frac{1}{2}\frac{1}{\sin^2\chi}(\overbrace{\frac{1}{R_\xi}}^{0}+\frac{1}{R_\phi}+2\frac{\overbrace{\cos\chi}^{0}}{R_{\xi\phi}}) = \frac{1}{2}\frac{1}{\sin^2\chi}\frac{1}{R_\phi}$$

$$= \frac{1}{2}\frac{6-2\cos\phi}{6-2\cos\phi-\sin^2\phi}\frac{2}{a\xi\sqrt{6-2\cos\phi-\sin^2\phi}}$$

$$= \frac{6-2\cos\phi}{(6-2\cos\phi-\sin^2\phi)^{3/2}}\frac{1}{a\xi}$$

$$K = \frac{1}{\sin^2\chi}(\frac{1}{R_\phi R_\xi}-\frac{1}{{R_{\phi\xi}}^2}) = 0$$

$$\frac{1}{R_1} = H+\sqrt{H^2-K} = 2H = \underline{\underline{\frac{2(6-2\cos\phi)}{(6-2\cos\phi-\sin^2\phi)^{3/2}}\frac{1}{a\xi}}},$$

$$\frac{1}{R_2} = H-\sqrt{H^2-K} = \underline{\underline{0}}.$$

$$\lambda_1 = \frac{A}{B}\frac{\overbrace{\frac{1}{R_{\xi\phi}}}^{0}+\frac{\cos\chi}{R_1}}{\frac{1}{R_\phi}-\frac{1}{R_1}} = \frac{A}{B}\frac{\cos\chi}{\frac{R_1}{R_\phi}-1} = \frac{1}{\xi}\frac{\sin\phi}{\frac{6-2\cos\phi-\sin^2\phi}{6-2\cos\phi}-1} = \underline{\underline{-\frac{1}{\xi}\frac{6-2\cos\phi}{\sin\phi}}}$$

$$\lambda_2 = \frac{A}{B}\frac{\overbrace{\frac{1}{R_{\xi\phi}}}^{0}+\frac{\overbrace{\cos\chi}^{0}}{R_2}}{\frac{1}{R_\phi}-\underbrace{\frac{1}{R_2}}_{0}} = \underline{\underline{0}}$$

(c)

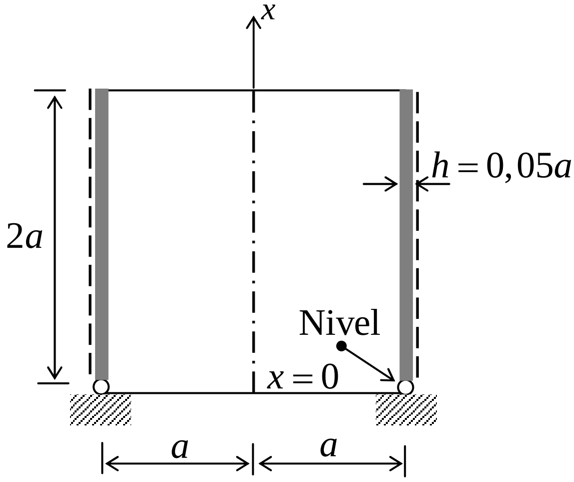
Kuori kokonaisuudessaan parabolinen.

Laskelma:

Piste	ϕ	ξ	A	B	$\cos\chi$	$\sin\chi$	$1/R_\phi$	$1/R_1$	λ_1	λ_2
P	0	1/2	$a/2$	$2a$	0	1	$\frac{2}{a}$	$\frac{2}{a}$	∞	0
Q	$\pi/2$	1/2	$a/2$	$a\sqrt{6}$	$-\frac{1}{\sqrt{6}}$	$\sqrt{\frac{5}{6}}$	$\frac{4}{a\sqrt{5}}$	$\frac{24\sqrt{5}}{25a}$	-12	0
R	π	1/2	$a/2$	$2a\sqrt{2}$	0	1	$\frac{\sqrt{2}}{a}$	$\frac{\sqrt{2}}{a}$	∞	0

Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävä 8:



Määritä oheisen sylinterikuoren normaalivoiman $N_\phi(x)$ ja taivutus-momentin $M_x(x)$ lausekkeet sen omasta painosta, kun kuoren tiheys on ρ, kimmomoduuli on E ja Poissonin vakio on $\nu = 0,3$. Piirrä dimensiottomat lausekkeet $N_\phi / (\rho g a^2)$ ja $M_x / (\rho g a^3)$ funktiona dimensiottomasta koordinaatista x / a. Määritä lopuksi näiden suureiden itseisarvoltaan suurimmat arvot.

Osittainen vastaus:

$$N_\phi(x) = -0,03\rho g a^2 e^{-5,7485\frac{x}{a}} \cos(5,7485\frac{x}{a})$$

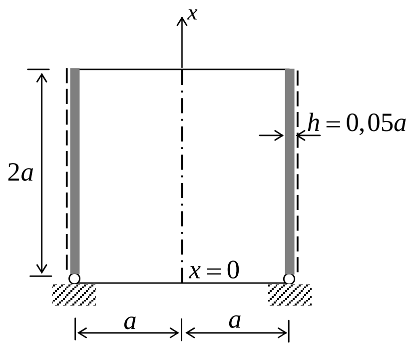
$$M_x(x) = 4,539 \cdot 10^{-4} \rho g a^3 e^{-5,7485\frac{x}{a}} \sin(5,7485\frac{x}{a})$$

$$N_{\phi,\min} = -0,03\rho g a^2,\ M_{x,\max} \approx 1,464 \cdot 10^{-4} \rho g a^3$$

Palautus pe 12.11 klo 16.00 mennessä.

Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävän 8 ratkaisu:



Kerroin β:

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{a^2h^2}} = \sqrt[4]{\frac{3(1-0,3^2)}{a^2(0,05a)^2}} = \frac{5,7485}{a}$$

$$\beta L = \frac{5,7485}{a} \cdot 2a = 11,497 > 5 \;\Rightarrow\; \text{Puoliääretön kuori}$$

Kuorma q_x:

$$q_x = -\rho g h = -0,05\rho g a$$

Normaalivoima N_x:

$$N_x = -\int q_x dx + C = 0,05\rho g a x + C$$

$$N_x(2a) = 0,05\rho g a \cdot 2a + C = 0 \;\Rightarrow\; C = -0,1\rho g a^2$$

$$\underline{N_x = 0,05\rho g a(x-2a)}$$

Muunnettu kuorma q_n^*:

$$q_n^* = \overbrace{q_n}^{0} - \frac{\overbrace{\nu}^{0,3}}{a} N_x = 0,015\rho g(2a-x)$$

Yksityisratkaisu:

$$w_0 = \frac{1}{4D\beta^4} q_n^* = \frac{12(1-\nu^2)}{4Eh^3\beta^4} \cdot 0,015\rho g(2a-x)$$

$$= \frac{12\cancel{(1-\nu^2)}}{4Eh^{\cancel{3}}\,\frac{3\cancel{(1-\nu^2)}}{a^2\cancel{h^2}}} \cdot 0,015\rho g(2a-x) = \underline{0,3\frac{\rho g a}{E}(2a-x)}$$

Ratkaisu integrointivakioiden avulla:

$$w(x) = e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + w_0(x)$$

$$= e^{-\beta x}(C_1 \cos\beta x + C_2 \sin\beta x) + 0,3\frac{\rho g a}{E}(2a-x)$$

$$\varphi(x) \equiv w'(x) = -\beta e^{-\beta x}[C_1(\cos\beta x + \sin\beta x) + C_2(\sin\beta x - \cos\beta x)] + w_0'(x)$$

$$= -\beta e^{-\beta x}[C_1(\cos\beta x + \sin\beta x) + C_2(\sin\beta x - \cos\beta x)] - 0,3\frac{\rho g a}{E}$$

$$M_x(x) \equiv -Dw''(x) = 2D\beta^2 e^{-\beta x}(-C_1 \sin\beta x + C_2 \cos\beta x) - D\overbrace{w_0''(x)}^{0}$$

$$= 2D\beta^2 e^{-\beta x}(-C_1 \sin\beta x + C_2 \cos\beta x)$$

Integrointivakiot reunaehtojen perusteella:

$$w(0) \equiv C_1 + 0,6\frac{\rho g a^2}{E} = 0 \;\Rightarrow\; \underline{C_1 = -0,6\frac{\rho g a^2}{E}}$$

$$M_x(0) \equiv 2D\beta^2 e^{-\beta x} C_2 = 0 \;\Rightarrow\; \underline{C_2 = 0}$$

Taipuma:

$$w(x) = -0,6\frac{\rho g a^2}{E} e^{-\beta x}\cos\beta x + 0,3\frac{\rho g a}{E}(2a-x)$$

Normaalivoima N_ϕ:

$$N_\phi(x) = \frac{Eh}{a} w(x) + \nu N_x(x)$$

$$= -0,03\rho g a^2 e^{-\beta x}\cos\beta x + 0,015 g a(2a - x) + 0,3\cdot 0,05\rho g a(x - 2a)$$

$$= -0,03\rho g a^2 e^{-\beta x}\cos\beta x = \underline{\underline{-0,03\rho g a^2 e^{-5,7485\frac{x}{a}}\cos(5,7485\frac{x}{a})}}$$

Taivutusmomentti M_x:

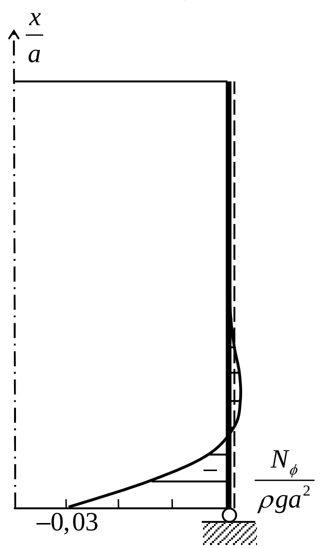
$$M_x(x) = 2D\beta^2 e^{-\beta x} 0,6\frac{\rho g a^2}{E}\sin\beta x$$

$$= 2\frac{Eh^{\cancel{3}}}{12\cancel{(1-\nu^2)}}\frac{3\cancel{(1-\nu^2)}}{\cancel{h^2}}\frac{e^{-\beta x}}{\beta^2 a^2}0,6\frac{\rho g a^2}{E}\sin\beta x = \frac{0,015}{\beta^2 a^2}\rho g a^3 e^{-\beta x}\sin\beta x$$

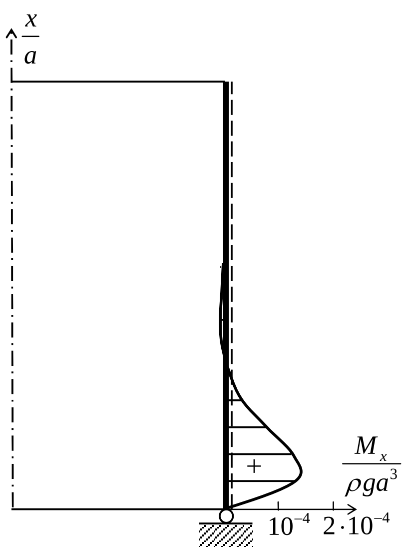
$$= 4,539\cdot 10^{-3}\rho g a^3 e^{-\beta x}\sin\beta x = \underline{\underline{4,539\cdot 10^{-4}\rho g a^3 e^{-5,7485\frac{x}{a}}\sin(5,7485\frac{x}{a})}}$$

Kuviot:

Rengasvoima N_ϕ:



Taivutusmomentti M_x:



Itseisarvoltaan suurimmat arvot:

$$N_{\phi,\min} = N_\phi(0) = \underline{\underline{-0,03\rho g a^2}}$$

$$M_x(x) = 4,539\cdot 10^{-3}\rho g a^3 e^{-5,7485\frac{x}{a}}\sin(5,7485\frac{x}{a}),$$

$$M_x'(x) = 5,7485\cdot 4,539\cdot 10^{-3}\rho g a^2 e^{-5,7485\frac{x}{a}}[-\sin(5,7485\frac{x}{a}) + \cos(5,7485\frac{x}{a})] = 0$$

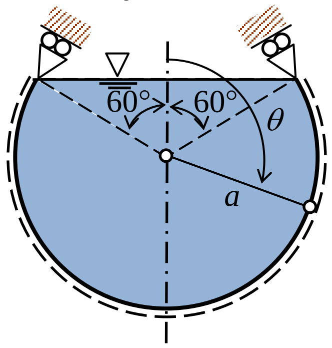
$$\Rightarrow\ \tan(5,7485\frac{x}{a}) = 1\ \Rightarrow\ 5,7485\frac{x}{a} = \frac{\pi}{4}\ \Rightarrow\ x = \frac{\pi}{4\cdot 5,7485}a \approx \underline{0,1366a}$$

$$M_{x,\max} \equiv M_x(0,1366a) = 4,539\cdot 10^{-3}\rho g a^3 e^{-\frac{\pi}{4}}\sin(\frac{\pi}{4}) \approx \underline{\underline{1,464\cdot 10^{-3}\rho g a^3}}$$

Rak-54.3100 Rakenteiden mekaniikka III, 2010

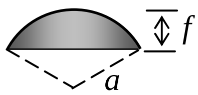
Kotitehtävä 9:

Osa (a): Määritä oheisen kalvotilassa olevan pallonmuotoisen nestesäiliön kalvovoimat $N_\theta(\theta)$ ja $N_\phi(\theta)$ sekä piirrä niiden kuvaajat. Nesteen tiheys on ρ, kuoren paksuus on $h = a/25$ ja Poissonin vakio on $\nu = 1/4$.



Ohje: Pallosegmentin tilavuuden kaava oheisen kuvan merkinnöin on

$$V = \frac{\pi}{3} f^2 (3a - f)$$

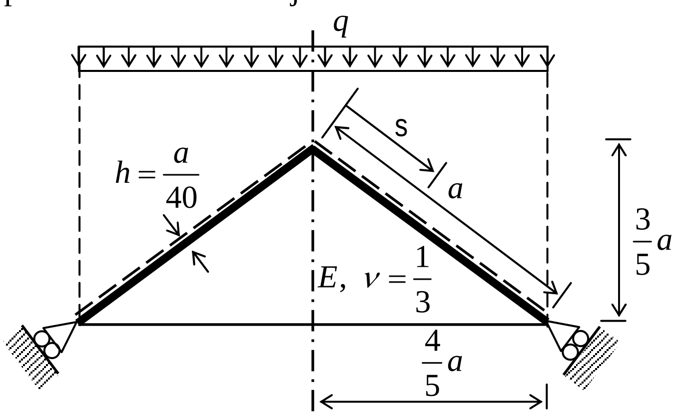


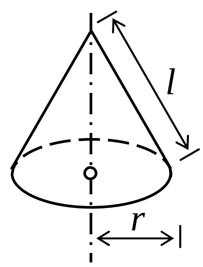
Vastaus:

$$N_\theta = \rho g a^2 \left[\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)} + \frac{1}{2}\left(\frac{1}{2} - \cos\theta\right)\right],$$

$$N_\phi = \rho g a^2 \left[-\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)} + \frac{1}{2}\left(\frac{1}{2} - \cos\theta\right)\right].$$

Osa (b): Määritä oheisen kalvotilassa olevan kartiokuoren muotoisen katon kalvovoimat $N_s(\mathrm{s})$ ja $N_\phi(\mathrm{s})$ (i) kuoren omasta painosta sekä (ii) vaakatason pintayksikköä kohti tasan jakautuneesta lumikuormasta q. Kuoren tiheys on ρ, sen paksuus on $h = a/40$ ja Poissonin vakio on $\nu = 1/3$.





Ohje: Kartion pinta-alan kaava oheisen kuvan merkinnöin on $A = \pi r l$.

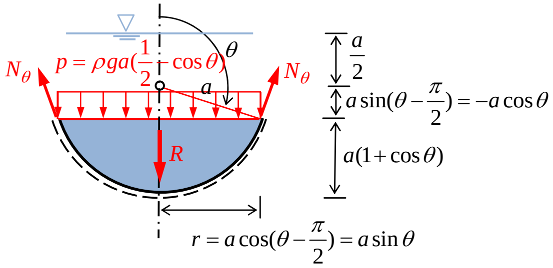
Vastaus: (i) $N_\mathrm{s} = -\frac{\rho g a}{48}\mathrm{s}$, $N_\phi = -\frac{2}{75}\rho g a \mathrm{s}$, (ii) $N_\mathrm{s} = -\frac{2}{3} q \mathrm{s}$, $N_\phi = -\frac{64}{75} q \mathrm{s}$.

Palautus pe 19.11 klo 16.00 mennessä.

Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävän 9 ratkaisu:

Osa (a):



$$\downarrow\quad -2\pi r N_\theta \sin\theta + R = 0 \;\Rightarrow -2\pi a\sin^2\theta\, N_\theta + R = 0 \;\Rightarrow\; N_\theta = \frac{R}{2\pi a\sin^2\theta}$$

$$R=\rho g V + pA = \rho g\cdot\frac{\pi}{3}[a(1+\cos\theta)]^2[3a-a(1+\cos\theta)]+\rho g a(\frac{1}{2}-\cos\theta)\cdot\pi r^2$$

$$=\rho g a^3\cdot\pi[\frac{1}{3}(1+\cos\theta)^2(2-\cos\theta)+(\frac{1}{2}-\cos\theta)\sin^2\theta]$$

$$N_\theta=\frac{1}{2\pi a\sin^2\theta}\cdot\rho g a^3\cdot\pi[\frac{1}{3}(1+\cos\theta)^2(2-\cos\theta)+(\frac{1}{2}-\cos\theta)\sin^2\theta]$$

$$=\rho g a^2[\frac{1}{6(1-\cos^2\theta)}(1+\cos\theta)^2(2-\cos\theta)+\frac{1}{2}(\frac{1}{2}-\cos\theta)]$$

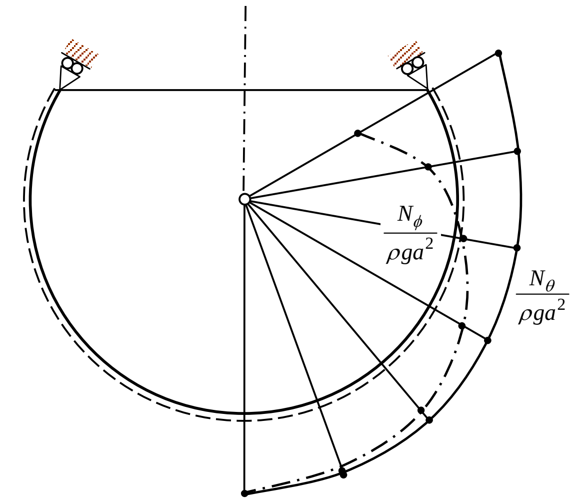
$$=\underline{\underline{\rho g a^2[\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)}+\frac{1}{2}(\frac{1}{2}-\cos\theta)]}}$$

$$N_\phi=-\frac{R_\phi}{R_\theta}N_\theta+q_n R_\phi=-N_\theta+pa$$

$$=-\rho g a^2[\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)}+\frac{1}{2}(\frac{1}{2}-\cos\theta)]+\rho g a^2(\frac{1}{2}-\cos\theta)$$

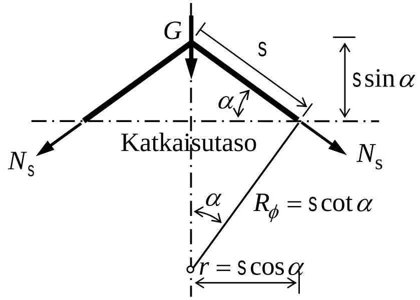
$$=\underline{\underline{\rho g a^2[-\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)}+\frac{1}{2}(\frac{1}{2}-\cos\theta)]}}$$

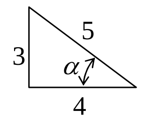
θ	$\frac{N_\theta}{\rho g a^2}$	$\frac{N_\phi}{\rho g a^2}$
60	$\frac{3}{4}=0,75$	$-\frac{3}{4}=-0,75$
80	0,595	$-0,269$
100	0,592	0,082
120	$\frac{23}{36}\approx 0,639$	$\frac{13}{36}\approx 0,361$
140	0,694	0,572
160	0,735	0,705
180	$\frac{3}{4}=0,75$	$\frac{3}{4}=0,75$



Osa (b):

(i)





$\sin\alpha=3/5$

$\cos\alpha=4/5$

$\tan\alpha=3/4$

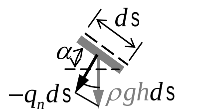
$$G=\rho g h A=\rho g\cdot\frac{a}{40}\cdot\pi r s=\frac{\pi}{40}\rho g a r s$$

$$\downarrow\; N_s\sin\alpha\cdot 2\pi r+G=0$$

$$\Rightarrow\; N_s=-\frac{G}{2\pi r\sin\alpha}=-\frac{1}{2\pi r\sin\alpha}\cdot\frac{\pi}{40}\rho g a r s=-\underline{\underline{\frac{\rho g a}{48}}}s$$

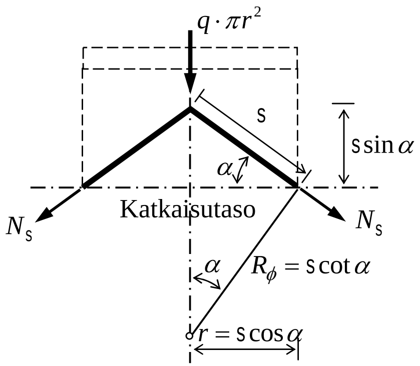
$$q_n=-\rho g h\cos\alpha=-\rho g\frac{a}{40}\frac{4}{5}=-\frac{\rho g a}{50}$$

$$N_\phi=q_n R_\phi=-\frac{\rho g a}{50}\cdot s\overbrace{\cot\alpha}^{4/3}=-\underline{\underline{\frac{2}{75}\rho g a s}}$$



$$-q_n ds=\rho g h\cos\alpha\, ds$$

(ii)

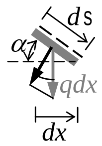


$$\downarrow\; N_\mathsf{s}\sin\alpha \cdot 2\pi r + q\cdot\pi r^2 = 0 \;\Rightarrow\; N_\mathsf{s} = -\frac{qr}{2\sin\alpha} = -\frac{q\mathsf{s}\cot\alpha}{2} = \underline{\underline{-\frac{2}{3}q\mathsf{s}}}$$

$$q_n = -q\cos^2\alpha = -\frac{16}{25}q$$

$$N_\phi = q_n R_\phi = \underline{\underline{-\frac{64}{75}q\mathsf{s}}}$$

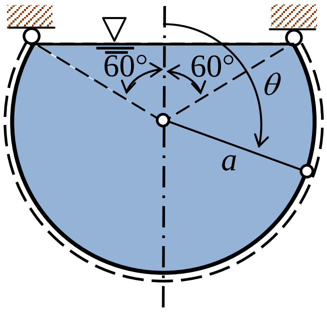
$$-q_n d\mathsf{s} = q\,dx\cos\alpha$$
$$= q\,d\mathsf{s}\cos^2\alpha$$



Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävä 10:

Osa (a): Määritä oheisen yläreunastaan nivelellisesti tuetun pallonmuotoisen nestesäiliön normaalivoiman $N_\phi(\theta)$ ja taivutusmomentin $M_\theta(\theta)$ lausekkeet, jälkimmäisen minimiarvo sekä piirrä niiden kuvaajat. Nesteen tiheys on ρ, kuoren paksuus on $h = a/25$ ja Poissonin vakio on $\nu = 1/4$.

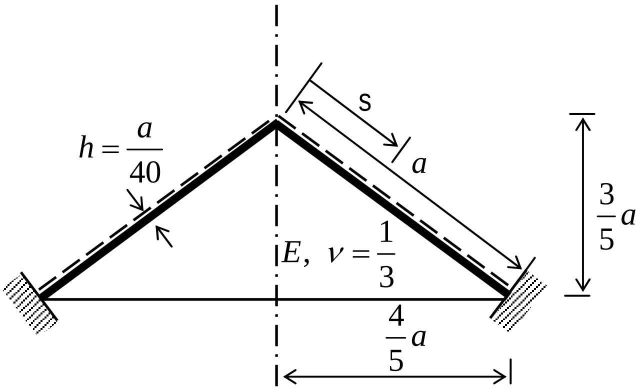


Vastaus:

$$N_\phi(\theta) = \rho g a^2 [-\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)} + \frac{1}{2}(\frac{1}{2} - \cos\theta) + 0{,}93753 e^{-6{,}4751(\theta - \frac{\pi}{3})} \cos 6{,}4751(\theta - \frac{\pi}{3})]$$

$$M_\theta(\theta) = -0{,}0111805 \rho g a^3 e^{-6{,}4751(\theta-\frac{\pi}{3})} \sin[6{,}4751(\theta - \frac{\pi}{3})],\ M_{\theta,\min} \approx -3{,}60 \cdot 10^{-3} \rho g a^3$$

Osa (b): Määritä oheisen alareunastaan jäykästi kiinnitetyn kartiokuoren muotoisen katon normaalivoiman $N_\phi(s)$ ja taivutusmomentin $M_s(s)$ lausekkeet sen omasta painosta sekä piirrä niiden kuvaajat. Kuoren tiheys on ρ, sen paksuus on $h = a/40$ ja Poissonin vakio on $\nu = 1/3$.



Vastaus:

$$N_\phi(s) = \rho g a s\{-\frac{2}{75} + e^{-8{,}0821(1-\frac{s}{a})}\{0{,}02147 \cos[8{,}0821(1-\frac{s}{a})] + 0{,}02901 \sin[8{,}0821(1-\frac{s}{a})]\}\}$$

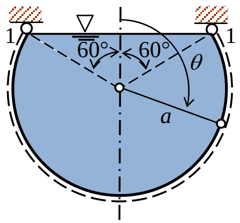
$$M_s(s) = \rho g a^3 e^{-8{,}0821(1-\frac{s}{a})}\{1{,}6656 \cdot 10^{-4} \cos[8{,}0821(1-\frac{s}{a})] - 1{,}2324 \cdot 10^{-4} \sin[8{,}0821(1-\frac{s}{a})]\}$$

Palautus pe 26.11 (viimeistään 10.12) klo 16.00 mennessä.

Rak-54.3100 Rakenteiden mekaniikka III, 2010

Kotitehtävän 10 ratkaisu:

Osa (a):



Kalvovoiman reunalla 1 (vrt. Kotitehtävä 9 osa (a)):

$$N_{\theta 1}^K \equiv N_\theta^K(60°) = \rho g a^2[\frac{(1+\cos 60°)(2-\cos 60°)}{6(1-\cos 60°)} + \frac{1}{2}(\frac{1}{2}-\cos 60°)] = \frac{3}{4}\rho g a^2,$$

$$N_{\phi 1}^K \equiv N_\phi^K(60°) = \rho g a^2[-\frac{(1+\cos 60°)(2-\cos 60°)}{6(1-\cos 60°)} + \frac{1}{2}(\frac{1}{2}-\cos 60°)] = -\frac{3}{4}\rho g a^2.$$

Kalvotilan venymä ε_ϕ reunalla 1:

$$\varepsilon_{\theta 1}^K = \frac{1}{Eh}(N_{\theta 1}^K - \nu N_{\phi 1}^K) = \frac{1}{Ea/25}(1+\frac{1}{4})\frac{3}{4}\rho g a^2 = \frac{375}{16}\frac{\rho g a}{E},$$

$$\varepsilon_{\phi 1}^K = \frac{1}{Eh}(N_{\phi 1}^K - \nu N_{\theta 1}^K) = \frac{1}{Ea/25}(-1-\frac{1}{4})\frac{3}{4}\rho g a^2 = -\frac{375}{16}\frac{\rho g a}{E}.$$

Kalvotilan taipuma reunalla 1 (vrt. Kotitehtävä 9 osa (a)):

$$w_1^K = w^K(60°) = -\overbrace{u(60°)}^{0}\cot(60°) + \overbrace{R_\phi(60°)}^{a}\varepsilon_\phi^K(60°) = a\cdot(-\frac{375}{16})\frac{\rho g a}{E} = -\frac{375}{16}\frac{\rho g a^2}{E}$$

Vaimennusluku ja taivutusjäykkyys:

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{h^2 R_\phi^2}} = \sqrt[4]{\frac{3[1-(\frac{1}{4})^2]}{(\frac{a}{25})^2\cdot a^2}} = \frac{6,4751}{a},$$

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{E(\frac{a}{25})^3}{12[1-(\frac{1}{4})^2]} = \frac{4}{703125}Ea^3 = 5,6889\cdot 10^{-6}Ea^3$$

Reunan leikkausvoiman määrittäminen:

$$w_1 \equiv -\frac{1}{2D\beta^3}Q_1 - \frac{1}{2D\beta^2}\overbrace{M_1}^{0} + w_1^K = 0 \;\Rightarrow$$

$$Q_1 = 2D\beta^3 w_1^K = 2\cdot\frac{4}{703125}Ea^3\cdot(\frac{6,4751}{a})^3\cdot(-\frac{375}{16}\frac{\rho g a^2}{E}) = \underline{-0,072395\rho g a^2}$$

Taivutusmomentti M_θ ja normaalivoima N_ϕ:

$$M_\theta \equiv M_\theta^T = \frac{e^{-\beta s}}{\beta}[\beta\overbrace{M_1}^{0}\cos\beta s + (Q_1 + \beta\overbrace{M_1}^{0})\sin\beta s] = \frac{Q_1}{\beta}e^{-\beta s}\sin\beta s$$

$$= \frac{-0,072395\rho g a^2}{6,4751/a}e^{-\beta s}\sin\beta s = -0,0111805\rho g a^3 e^{-\beta s}\sin\beta s$$

$$N_\phi \equiv N_\phi^K + N_\phi^T = N_\phi^K + 2R_\phi\beta e^{-\beta s}[(-Q_1 - \beta\overbrace{M_1}^{0})\cos\beta s + \beta\overbrace{M_1}^{0}\sin\beta s] = N_\phi^K - 2R_\phi\beta e^{-\beta s}Q_1\cos\beta s$$

$$= \rho g a^2[-\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)} + \frac{1}{2}(\frac{1}{2}-\cos\theta)] - 2a\frac{6,4751}{a}e^{-\beta s}\cos\beta s\cdot(-0,072395\rho g a^2)$$

$$= \rho g a^2[-\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)} + \frac{1}{2}(\frac{1}{2}-\cos\theta) + 0,93753e^{-\beta s}\cos\beta s]$$

Käyttäen yhteyttä

$$s = (\theta - \frac{\pi}{3})a\,,$$

saadaan näistä

$$M_\theta = \underline{\underline{-0,0111805\rho g a^3 e^{-6,4751(\theta-\frac{\pi}{3})}\sin[6,4751(\theta-\frac{\pi}{3})]}}$$

$$N_\phi = \underline{\underline{\rho g a^2[-\frac{(1+\cos\theta)(2-\cos\theta)}{6(1-\cos\theta)} + \frac{1}{2}(\frac{1}{2}-\cos\theta) + 0,93753e^{-6,4751(\theta-\frac{\pi}{3})}\cos 6,4751(\theta-\frac{\pi}{3})]}}$$

Taivutusmomentin maksimiarvo:

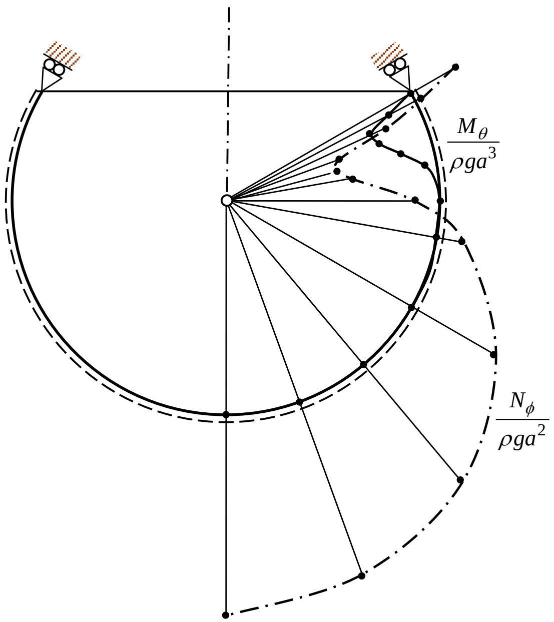
$$\frac{dM_\theta}{d\theta} \equiv -0,0111805\rho g a^3 e^{-6,4751(\theta-\frac{\pi}{3})}\{-\sin[6,4751(\theta-\frac{\pi}{3})] + \cos[6,4751(\theta-\frac{\pi}{3})\} = 0$$

$$\Rightarrow \tan[6,4751(\theta-\frac{\pi}{3})] = 1 \;\Rightarrow\; 6,4751(\theta-\frac{\pi}{3}) = \frac{\pi}{4} \;\Rightarrow\; \theta_{\max} = \frac{\pi}{3} + \frac{\pi}{4\cdot 6,4751} \approx 1,168$$

$$M_{\theta,\min} = -0,0111805\rho g a^3 e^{-6,4751(1,168-\frac{\pi}{3})}\sin[6,4751(1,168-\frac{\pi}{3})] \approx \underline{\underline{-3,60\cdot 10^{-3}\rho g a^3}}$$

Lasketaan arvoja:

θ [°]	$M_\theta/(\rho g a^3)$	$N_\phi/(\rho g a^2)$
60	0	0,188
62	$-1,20\cdot 10^{-3}$	0,038
65	$-2,04\cdot 10^{-3}$	-0,159
70	$-1,96\cdot 10^{-3}$	-0,355
75	$-1,22\cdot 10^{-3}$	-0,394
80	$-0,54\cdot 10^{-3}$	-0,331
90	$0,06\cdot 10^{-3}$	-0,114
100	$0,07\cdot 10^{-3}$	0,080
120	0	0,362
140	0	0,572
160	0	0,705
180	0	0,750



Osa (b):

Kalvovoimat (vrt. Kotitehtävä 9 osa (b)):

$$N_{\mathsf{s}}^K = -\frac{\rho g a}{48}\mathsf{s},\ N_\phi^K = -\frac{2}{75}\rho g a\mathsf{s}.$$

Kalvotilan venymät ε_{s} ja ε_ϕ sekä derivaatta $d\varepsilon_\phi / d\mathsf{s}$:

$$\varepsilon_{\mathsf{s}}^K = \frac{1}{Eh}(N_{\mathsf{s}}^K - \nu N_\phi^K) = \frac{40}{Ea}[-\frac{\rho g a}{48}\mathsf{s} - \frac{1}{3}(-\frac{2}{75}\rho g a\mathsf{s})] = -\frac{43}{90}\frac{\rho g}{E}\mathsf{s},$$

$$\varepsilon_{\phi 2}^K = \frac{1}{Eh}(N_\phi^K - \nu N_{\mathsf{s}}^K) = \frac{40}{Ea}[-\frac{2}{75}\rho g a\mathsf{s} - \frac{1}{3}(-\frac{\rho g a}{48}\mathsf{s})] = -\frac{71}{90}\frac{\rho g}{E}\mathsf{s},$$

$$\frac{d\varepsilon_\phi^K}{d\mathsf{s}} = -\frac{71}{90}\frac{\rho g}{E}.$$

Kalvotilan venymät ε_{s} ja ε_ϕ sekä derivaatta $d\varepsilon_\phi / d\mathsf{s}$ reunalla 2:

$$\varepsilon_{\mathsf{s}2}^K \equiv \varepsilon_{\mathsf{s}}^K(a) = -\frac{43}{90}\frac{\rho g a}{E},\ \varepsilon_{\phi 2}^K \equiv \varepsilon_\phi^K(a) = -\frac{71}{90}\frac{\rho g a}{E},\ \frac{d\varepsilon_{\phi 2}^K}{d\mathsf{s}} \equiv \frac{d\varepsilon_\phi^K}{d\mathsf{s}}(a) = -\frac{71}{90}\frac{\rho g}{E}.$$

Kalvotilan taipuma ja kiertymä reunalla 2:

$$w_2^K = w^K(a) = -\overbrace{u(a)}^{0}\cot\theta + \overbrace{R_\phi(a)}^{a\cot\alpha}\varepsilon_{\phi 2}^K = a\overbrace{\cot\theta}^{4/3}\cdot(-\frac{71}{90}\frac{\rho g a}{E}) = -\frac{142}{135}\frac{\rho g a^2}{E}$$

$$\varphi_{\mathsf{s}2}^K = \overbrace{\cot\theta}^{4/3}(\varepsilon_{\phi 2}^K - \varepsilon_{\mathsf{s}2}^K) + \overbrace{R_\phi(a)}^{4/3a}\frac{d\varepsilon_\phi^K}{d\mathsf{s}} = \frac{4}{3}[-\frac{71}{90}\frac{\rho g a}{E} + \frac{43}{90}\frac{\rho g a}{E}] + \frac{4}{3}a(-\frac{71}{90}\frac{\rho g a^2}{E}) = -\frac{22}{15}\frac{\rho g a}{E}$$

Vaimennusluku ja taivutusjäykkyys:

$$\beta = \sqrt[4]{\frac{3(1-\nu^2)}{h^2 R_\phi^2}} = \sqrt[4]{\frac{3[1-(\frac{1}{3})^2]}{(\frac{a}{40})^2\cdot a^2}} = \frac{8,0821}{a},$$

$$D = \frac{Eh^3}{12(1-\nu^2)} = \frac{E(\frac{a}{40})^3}{12[1-(\frac{1}{3})^2]} = \frac{3}{2048000}Ea^3 = 1,4648\cdot 10^{-6}Ea^3$$

Reunan leikkausvoiman ja taivutusmomentin määrittäminen:

$$\left.\begin{aligned} w_2 &\equiv \frac{1}{2D\beta^3}Q_2 - \frac{1}{2D\beta^2}M_2 + w_2^K = 0 \\ \varphi_2 &\equiv \frac{1}{2D\beta^2}Q_2 - \frac{1}{D\beta}M_2 + \varphi_2^K = 0 \end{aligned}\right\}$$

$$\Rightarrow \begin{cases} Q_2 - \beta M_2 = -2D\beta^3 w_2^K \\ Q_2 - 2\beta M_2 = -2D\beta^2\varphi_2^K \end{cases} \Rightarrow \begin{cases} Q_2 = 2D\beta^2(\varphi_2^K - 2\beta w_2^K), \\ M_2 = 2D\beta(\varphi_2^K - \beta w_2^K). \end{cases}$$

$$\Rightarrow \begin{cases} Q_2 = 2\cdot 1,4648\cdot 10^{-6}Ea^3\cdot(\frac{8,0821}{a})^2\cdot[-\frac{22}{15}\frac{\rho g a}{E} - 2\cdot\frac{8,0821}{a}(-\frac{142}{135}\frac{\rho g a^2}{E})], \\ M_2 = 2\cdot 1,4648\cdot 10^{-6}Ea^3\cdot\frac{8,0821}{a}\cdot[-\frac{22}{15}\frac{\rho g a}{E} - \frac{8,0821}{a}(-\frac{142}{135}\frac{\rho g a^2}{E})]. \end{cases}$$

$$\Rightarrow \begin{cases} Q_2 = 2,3422\cdot 10^{-3}\rho g a^2, \\ M_2 = 1,6656\cdot 10^{-4}\rho g a^3. \end{cases}$$

Normaalivoiman N_ϕ ja taivutusmomentin M_s lausekkeet:

$$N_\phi^T = N_\phi^K + N_\phi^T = N_s^K = -\frac{2}{75}\rho g a \mathsf{s} + 2R_\phi \beta e^{-\beta s}[(Q_2 - \beta M_2)\cos\beta s + \beta M_2 \sin\beta s]$$

$$= -\frac{2}{75}\rho g a \mathsf{s} + 2\cdot \mathsf{s}\overbrace{\cot\alpha}^{4/3}\cdot\frac{8,0821}{a}e^{-\beta s}[(2,3422\cdot 10^{-3}\rho g a^2 - \frac{8,0821}{a}\cdot 1,6656\cdot 10^{-4}\rho g a^3)\cos\beta s$$

$$+\frac{8,0821}{a}\cdot 1,6656\cdot 10^{-4}\rho g a^3 \sin\beta s]$$

$$= \rho g a \mathsf{s}[-\frac{2}{75} + e^{-\beta s}(0,02147\cos\beta s + 0,02901\sin\beta s)]$$

$$M_\mathsf{s} = M_\mathsf{s}^T = \frac{e^{-\beta s}}{\beta}[\beta M_2\cos\beta s + (-Q_2 + \beta M_2)\sin\beta s] = e^{-\beta s}[M_2\cos\beta s + (-\frac{Q_2}{\beta} + M_2)\sin\beta s]$$

$$= e^{-\beta s}[1,6656\cdot 10^{-4}\rho g a^3\cos\beta s + (-\frac{2,3422\cdot 10^{-3}\rho g a^2\cdot a}{8,0821} + 1,6656\cdot 10^{-4}\rho g a^3)\sin\beta s]$$

$$= \rho g a^3 e^{-\beta s}(1,6656\cdot 10^{-4}\cos\beta s - 1,2324\cdot 10^{-4}\sin\beta s)$$

Käyttämällä lopulta yhteyttä $s = a - \mathsf{s}$ saadaan

$$N_\phi(\mathsf{s}) = \rho g a \mathsf{s}\{-\frac{2}{75} + e^{-8,0821(1-\frac{\mathsf{s}}{a})}\{0,02147\cos[8,0821(1-\frac{\mathsf{s}}{a})] + 0,02901\sin[8,0821(1-\frac{\mathsf{s}}{a})]\}\}$$

$$M_\mathsf{s}(\mathsf{s}) = \rho g a^3 e^{-8,0821(1-\frac{\mathsf{s}}{a})}\{1,6656\cdot 10^{-4}\cos[8,0821(1-\frac{\mathsf{s}}{a})] - 1,2324\cdot 10^{-4}\sin[8,0821(1-\frac{\mathsf{s}}{a})]\}$$

Lasketaan arvoja:

$\frac{\mathsf{s}}{a}$	$\frac{N_\phi}{\rho g a^2}$	$\frac{M_\mathsf{s}}{\rho g a^3}$
0	0	$-0,00\cdot 10^{-4}$
0,1	$-0,0027$	$-0,00\cdot 10^{-4}$
0,2	$-0,0053$	$0,00\cdot 10^{-4}$
0,3	$-0,0080$	$0,01\cdot 10^{-4}$
0,4	$-0,0107$	$0,01\cdot 10^{-4}$
0,5	$-0,0136$	$-0,00\cdot 10^{-4}$
0,6	$-0,0166$	$-0,06\cdot 10^{-4}$
0,7	$-0,0185$	$-0,18\cdot 10^{-4}$
0,8	$-0,0169$	$-0,26\cdot 10^{-4}$
0,85	$-0,0139$	$-0,17\cdot 10^{-4}$
0,90	$-0,0096$	$0,12\cdot 10^{-4}$
0,95	$-0,0056$	$0,70\cdot 10^{-4}$
0,975	$-0,0046$	$1,13\cdot 10^{-4}$
1	$-0,0052$	$1,67\cdot 10^{-4}$